Midterm Exam

Brown University CS145: Probability for Computing and Data Analysis  
October 24, 2019

Write both your name and your Banner ID on the cover of your solution booklet. To receive maximal credit for partially correct answers, please be sure to clearly show your work.

Answer any 3 out of the following 4 questions. If you answer all 4 questions, an arbitrary (possibly worst) three answers will be graded.

Question 1:
You throw darts at a circular target of radius $r$. You are equally likely to hit any point in the target. Let $X$ be the distance you hit from the center.

a) Compute the Cumulative Distribution Function and the Probability Density Function of $X$.

b) Compute the expectation and variance of $X$.

Question 2:
Joe tells Dan that he will let Dan roll $n$ fair, 6-sided dice. If a 6 shows up on any of them, Dan gets nothing. If no 6’s appear, Dan is paid the sum of the values on the dice in dollars. Dan is free to choose $n$, the number of dice rolls.

a) What is the smallest value of $n$ that maximizes Dan’s expected payoff?

b) Suppose Dan chooses to roll 10 dice (this is not necessarily the answer to part (a)). What is the expected number of distinct dice values that show up? In other words, what is the expected number of faces that are rolled at least once?

Question 3:
In some universities students can retake final exams for a course. You pass the course once you pass the final exam. Assume that your probability of passing the exam in each trial is $2/3$ and independent of previous trials.

a) What is the expected number of exams you take?

b) Assuming you do not pass the first $k$ final exams, what is the expected number of additional exams that you took?

c) Assuming you can only take up to $k$ final exams, what is the expected number of exams that you took?
Question 4:

The amount of traffic in winter in Providence depends on whether it snows or not, and it snows with probability \( \frac{1}{3} \). With no snow, the time for a taxi to drive from Brown to the airport is exponentially distributed with a mean of 15 minutes. With snow, the travel time is exponentially distributed with a mean of 25 minutes. Given that it takes you \( x \geq 0 \) minutes to reach the airport, what is the probability that it is snowing as a function of \( x \)?