Question 1

(a) Because there are 2 red aces (of diamonds and hearts) in a deck of 52 cards, we have
\[ P(\text{Card is a Red Ace}) = \frac{2}{52} = \frac{1}{26} \approx 0.038. \]

(b) Because 26 of the 52 cards in the deck are black, and each draw of a black card reduces the number of remaining black cards in the deck by one, we have
\[ P(\text{First 3 Cards are Black}) = \frac{26}{52} \times \frac{25}{51} \times \frac{24}{50} = \frac{15,600}{132,600} \approx 0.1176. \]

(c) Each 5-card hand is an unordered choice of 5 cards from the full deck of 52, so the number of 5-card hands equals
\[ \binom{52}{5} = \frac{52!}{47! \cdot 5!} = 2,598,960. \]

(d) Solution 1:
The number of 5-card hands containing at least one ace is the total number of 5-card hands minus the number of 5-card hands with no aces.
The number of 5-card hands with no aces is the number of 5-card hands from a deck without aces i.e. a 48-card deck. Thus we have
\[ \binom{52}{5} - \binom{48}{5} = \frac{52!}{47! \cdot 5!} - \frac{48!}{43! \cdot 5!} = 2,598,960 - 1,712,304 = 886,656 \]

Solution 2:
Alternatively we can add up the number of hands with 1, 2, 3 and 4 aces.
\[ \sum_{a=1}^{4} \binom{4}{a} \binom{48}{5-a} = 886,656 \]

Question 2

(a) Each draw could produce any of the 6 marbles, and thus there are \(6^3 = 216\) outcomes.

(b) There are 6 marbles that could be drawn first, and one fewer marbles for each subsequent draw, so the total number of possible outcomes equals \(6 \times 5 \times 4 = 120\).

(c) The probability of drawing at least 1 yellow marble is one minus the probability that no yellow marbles are drawn, which equals \(1 - \left(\frac{5}{6}\right)^3 = \frac{91}{216} \approx 0.421\).
Question 3

(a) Every sequence of 10 fair coin flips has equal probability $2^{-10}$. The number of sequences in which $k$ of these are heads is $\binom{10}{k}$, and thus

$$P(H = k) = \binom{10}{k} \cdot \left(\frac{1}{2}\right)^{10} \text{ for } k \in 0, 1, \ldots, 10.$$  

Clearly the probability that $H$ is greater than 10, or less than 0, equals zero.

(b) See the function `binomialPlots.m`, and the plot in Figure 1.

(c) A sequence of 10 biased coin flips with $k$ heads, and $10 - k$ tails, has probability $\binom{9}{k} \left(\frac{1}{10}\right)^{10-k} \cdot \left(\frac{1}{10}\right)^{10-k}$. Summing over the $\binom{10}{k}$ sequences with $k$ heads, the overall probability of the total number of heads $H$ is then

$$P(H = k) = \binom{10}{k} \cdot \left(\frac{9}{10}\right)^k \cdot \left(\frac{1}{10}\right)^{10-k} \text{ for } k \in 0, 1, \ldots, 10.$$  

See the function `binomialPlots.m`, and the plot in Figure 1.