Homework 1: Probability, Conditioning, & Bayes Rule

Brown University CS145: Probability & Computing

Homework due at 11:59pm on February 11, 2016

For all questions, simply giving the appropriate probability is insufficient. You must also clearly show the steps needed to derive your answer.

Question 1:

The CS department decides to raffle off a new laptop to a lucky CS145 student. Because they expect that younger students already have newer laptops, they decide to weight the raffle in favor of more senior students. In particular, the junior and senior classes each have a 1/3 chance of being chosen, the sophomore class has a 2/9 chance of being chosen, and the freshmen class only has a 1/9 chance of being chosen. Within the chosen class, all students have an equal chance of receiving the laptop. Suppose the counts of enrolled students are:

<table>
<thead>
<tr>
<th>Class</th>
<th># of males</th>
<th># of females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freshmen</td>
<td>17</td>
<td>19</td>
</tr>
<tr>
<td>Sophomores</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>Juniors</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Seniors</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

What is the probability that a female student wins the laptop?

Question 2:

A recent study has shown that 10% of computer science concentrators suffer from symptoms of Carpal Tunnel Syndrome. Researchers have developed a diagnostic test for this condition and in a recent trial, the test gave a positive result in 96.8% of the patients who were known to have Carpal Tunnel Syndrome. The researchers believe their test will be practically useful if a positive result implies the person has Carpal Tunnel with at least 75% probability. What is the largest possible false positive rate (the percentage of people without Carpal Tunnel who incorrectly test positive) that would achieve this accuracy target?

Question 3:

A group of 5 computer science concentrators are working together on a homework assignment that allows collaboration, and are stuck on a true/false question. They decide to each come up with their own answer independently, share their answers, and then choose whichever answer is in the majority. Suppose each student has a 75% chance of guessing correctly. What is the probability that the group decides on the correct answer?
Question 4:

According to a recent poll, the unemployment rate for college graduates in the United States is 2.9%. In the same period, the percentage of college graduates with a degree in computer science rose to 2.5%. For these computer science graduates, the unemployment rate is 4.9%.

a) Draw a Venn diagram summarizing the relevant variables. Associate each of the three probabilities provided above with some event in this diagram.

b) What percent of college graduates have a computer science degree and are employed?

c) What percent of college graduates have a computer science degree or are unemployed?

d) What percent of college graduates don’t have a computer science degree and are employed?

Question 5:

David conducts an experiment in which he rolls a fair six-sided die. If he rolls any number other than a 6, he records the result and rolls the die again. If he rolls a 6, he records the result and stops. If David rolls the die 8 times without rolling a 6, he becomes bored and stops anyway. To show your work below, submit your Matlab code as well as any derivations.

a) What is the probability that David rolls at least one 6? Derive an equation for this probability, and also compute its numeric value.

b) Use Matlab to determine the probability that David rolls at least one 5 by counting the number of outcomes where this occurs. Hint: You can verify your code by hand-checking the result for a smaller maximum number of die rolls.

c) Use Matlab to determine the probability that David rolls more 4’s than 3’s.

d) Clearly, eight dice rolls can sum to at most 48. Compute and plot the probabilities that the sum of David’s dice equals each integer between 1 and 48.

e) Erik rolls dice similarly, but stops if he rolls a 1 or after 8 rolls. Compute and plot the probability that the sum of Erik’s dice equals each integer between 1 and 48.

f) David and Erik play a game in which they each roll dice as described above, and the winner is the person who rolls the sum closest to some target integer $x$, without going over. If they roll the same sum, or both go over, no one wins. Compute and plot the probability that David wins, and the probability that Erik wins, for all targets $1 \leq x \leq 48$. 