Question 1

Two points $X$ and $Y$ are chosen independently and uniformly at random from the interval $[0, 1]$. What is the expected distance between the points?

Answer 1
Question 2

We bought a new lightbulb that will burn out after $t$ hours, where $t \geq 0$ is a real number with an exponential density

$$f(t) = \lambda e^{-\lambda t}$$

where $\lambda = .05$.

(a) What is the probability that the lightbulb will not burn out before $T$ hours?

(b) How many hours will it take for the lightbulb to have a $1/2$ chance of burning out?

(c) Given that the bulb lasted 20 hours, what is the probability that the bulb burns out before 21 hours?

(d) Given that the bulb lasted 20 hours, what is the probability that the bulb lasts at least 40 hours?

Answer 2
Question 3

Alice is sending Bob yes/no messages, encoded as 1 and −1, respectively.

However, the telephone line Alice is using adds noise to the value of her bit. The noise is normally distributed with mean $\mu = 0$ and variance 4. To account for the noise, the receiver returns 1 if the message is positive and -1 if negative.

What is the probability that the receiver makes an error on a single bit?

Answer 3
In this problem we use inverse transform sampling to randomly sample from an exponential distribution.

The inverse of a cumulative distribution function is sometimes called a “quantile function.” Given an input $x$, the quantile function for a distribution tells us the value of a random variable of that distribution such that the probability of the variable being less than or equal to that value equals $x$.

(a) Derive the inverse of the exponential CDF $1 - e^{-\lambda x}$.

(b) Using Matlab (or another language of your choice), write a function `exponential_sample` that generates a random value from a uniform distribution and uses the equation from part (a) to transform it into a random value from an exponential distribution. (To sample from a uniform distribution, you may use Matlab’s `rand` function).

(c) Generate 10,000 random values and use a histogram to compare their distribution to the exponential CDF. Do they match?

**Answer 4**