Question 1

A box contains 7 blue balls and 4 red balls. Two balls are drawn one after the other.

1. What is the probability that the second ball is red?

2. If the first ball is red, a blue ball is placed in the box, and if the first ball is blue, a red ball is placed in the box. The original ball is not returned to the box. Then a second ball is drawn. What is the probability the second ball is red?

3. If the first ball is blue, $m$ blue balls are added to the box. If the first ball is red, $m$ red balls are added to the box. The original ball is returned to the box. Then, a second ball is drawn. Show that the probability the second ball is red remains the same regardless of the value of $m$.

Answer
Question 2

(a) For two events $A$ and $B$ in a probability space, prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Hint: The part of $A$ that is not contained in $B$ can be written as $A \setminus B$.

(b) In parts (b), (c), and (d) we will walk through a proof by induction. For an event $A_1$, we know that $P(A_1) \geq P(A_1)$. Show that for events $A_1, A_2$, $P(A_1) + P(A_2) \geq P(A_1 \cup A_2)$.

(c) Assume $\sum_{i=1}^{n} P(A_i) \geq P(\bigcup_{i=1}^{n} A_i)$. Show for $n + 1$, $\sum_{i=1}^{n+1} P(A_i) \geq P(\bigcup_{i=1}^{n+1} A_i)$

(d) For any $n$ events $A_1, A_2, ..., A_n$ in a probability space, conclude that

$$P(A_1) + P(A_2) + ... + P(A_n) \geq P(A_1 \cup A_2 \cup ... \cup A_n)$$

Answer
Question 3

In a poker game, Claire has a very strong hand. The probability that Marie has a better hand is 0.06. If Marie had a better hand she would increase her bet (raise) with probability 0.8, but with a poorer hand she would only raise with probability 0.2.
If Marie doesn’t increase her bet, what is the probability that she has a better hand than Claire does?

Answer
Question 4

(a) Write a program that simulates the birthday problem by counting how many times two or more people have the same birthday in CS 1450 for \(2 \leq n \leq 80\) students for 10000 trials. Assume no one has a birthday on a leap year.

(b) Plot these probabilities on a graph. If there are 30 people in the class, what is the probability that two people share a birthday?

(c) Write a program that counts how many times a student shares Eli’s birthday, for \(2 \leq n \leq 80\) people (\(n\) includes Eli) for 10000 trials.

(d) Plot these probabilities on the same graph as in (b). If there are 30 people in the class, what is the probability that a person shares a birthday with Eli? Provide an intuitive explanation for why this answer is different than that of (b).

(e) On another planet (far, far away), a class of super-intelligent aliens is also taking CS 1450. There are 25 aliens in the class, but there are \(N\) days in their year and no leap years. Write a program that simulates the birthday problem by counting how many times two or more aliens share a birthday in CS 1450 for \(365 < N < 500\) days in the year for 10000 trials.

(f) Plot these probabilities on a graph. What is the largest number of days in a year such that there is a more than 0.5 chance that at least two aliens share a birthday? NOTE: A rough estimate is okay for this part.

Answer