Model Fitting

Computer Vision

CS 143, Brown

James Hays

Slides from Silvio Savarese, Svetlana Lazebnik, and Derek Hoiem

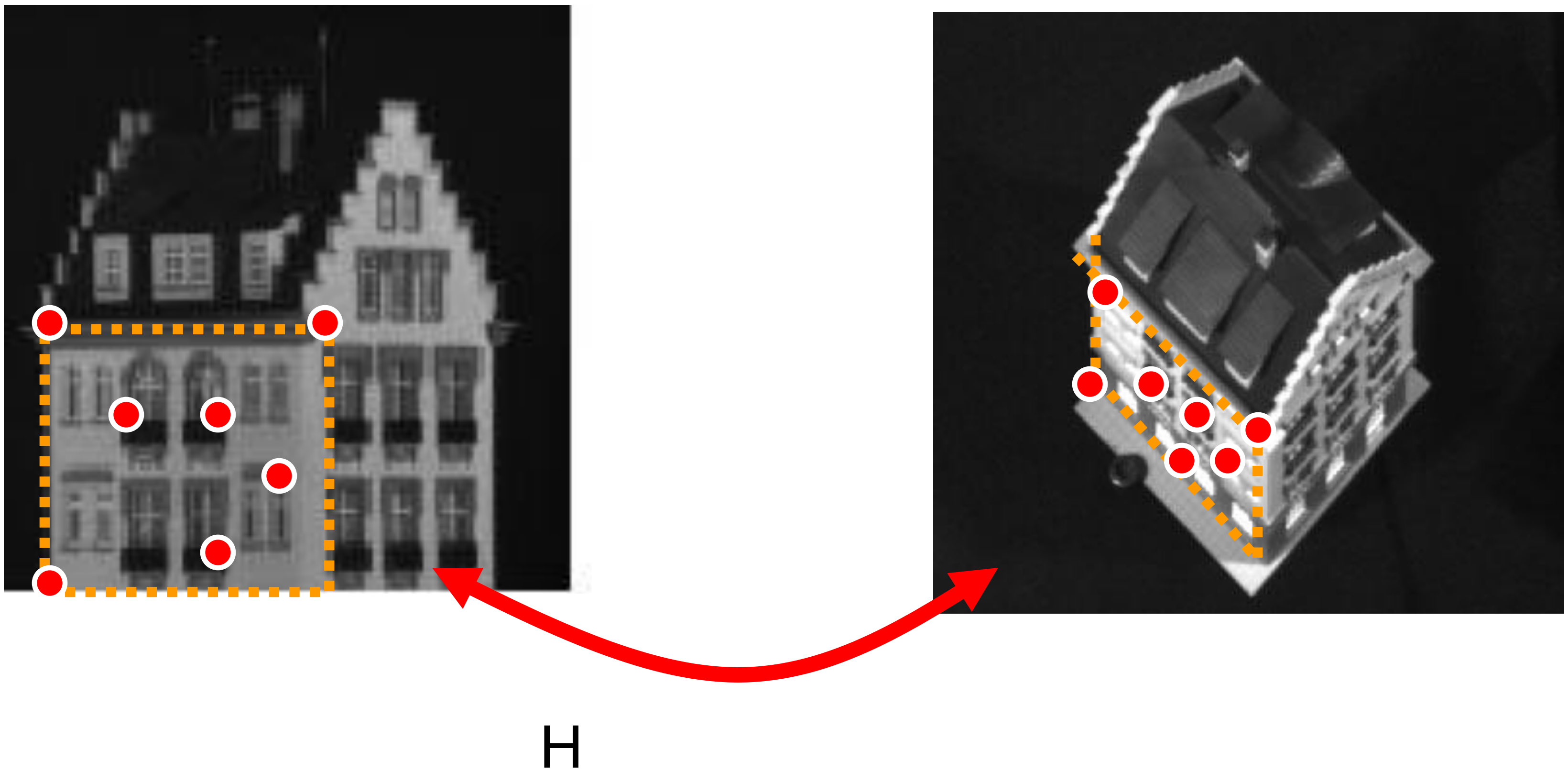
Fitting: find the parameters of a model that best fit the data

Alignment: find the parameters of the transformation that best align matched points

Example: Computing vanishing points

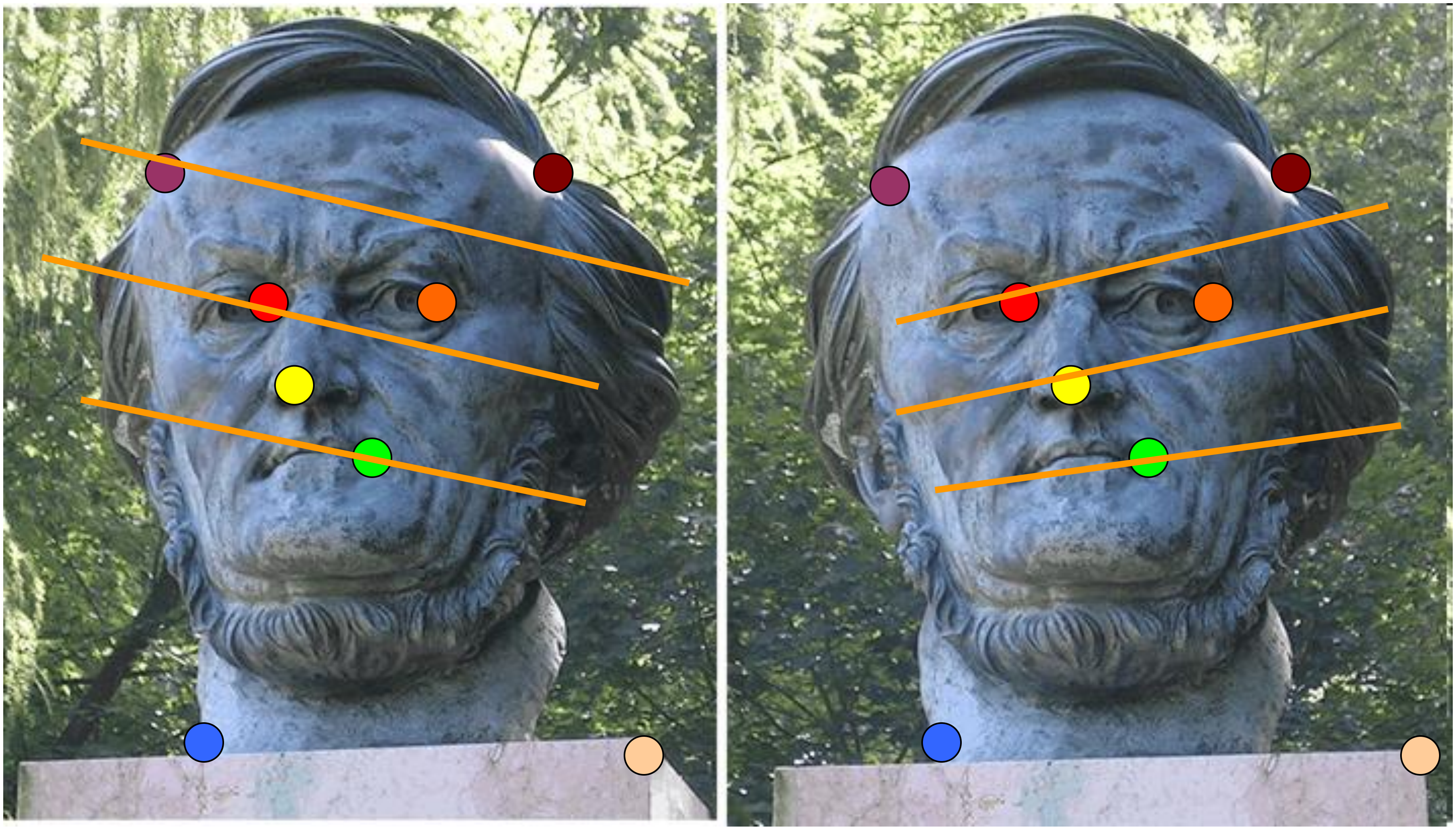
Slide from Silvio Savarese

Example: Estimating an homographic transformation



Slide from Silvio Savarese

Example: Estimating "fundamental matrix" that corresponds two views



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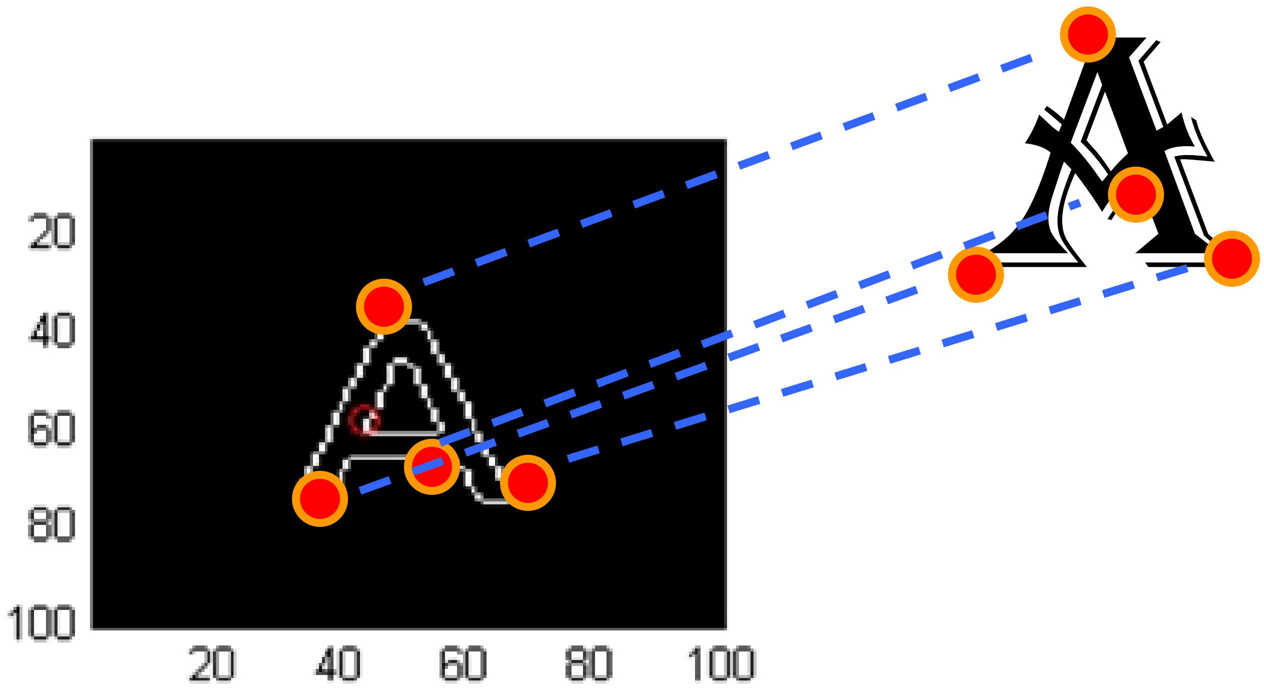
Example: fitting an 2D shape template



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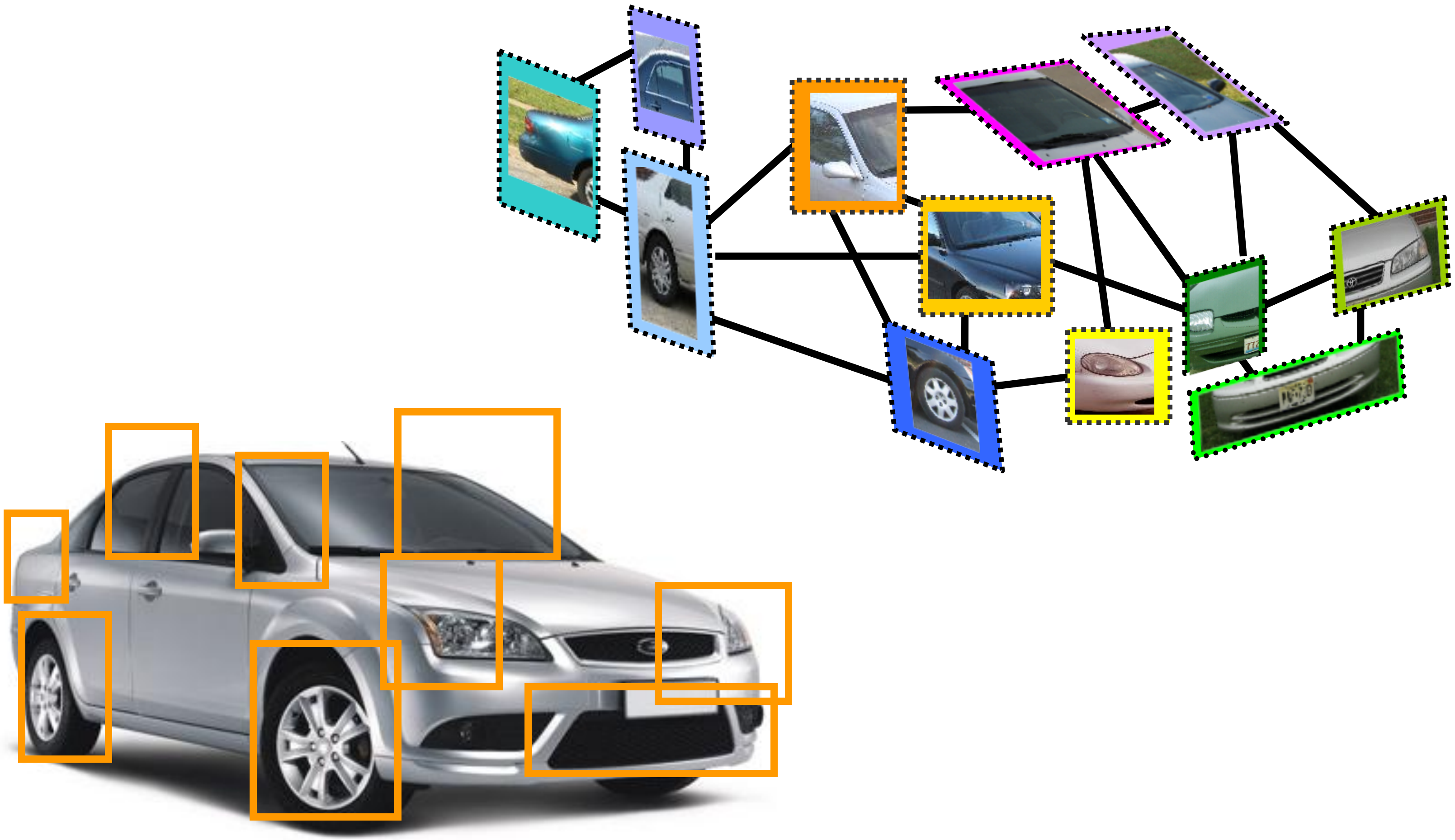
Example: fitting a 3D object model



Slide from Silvio Savarese

Critical issues: noisy data

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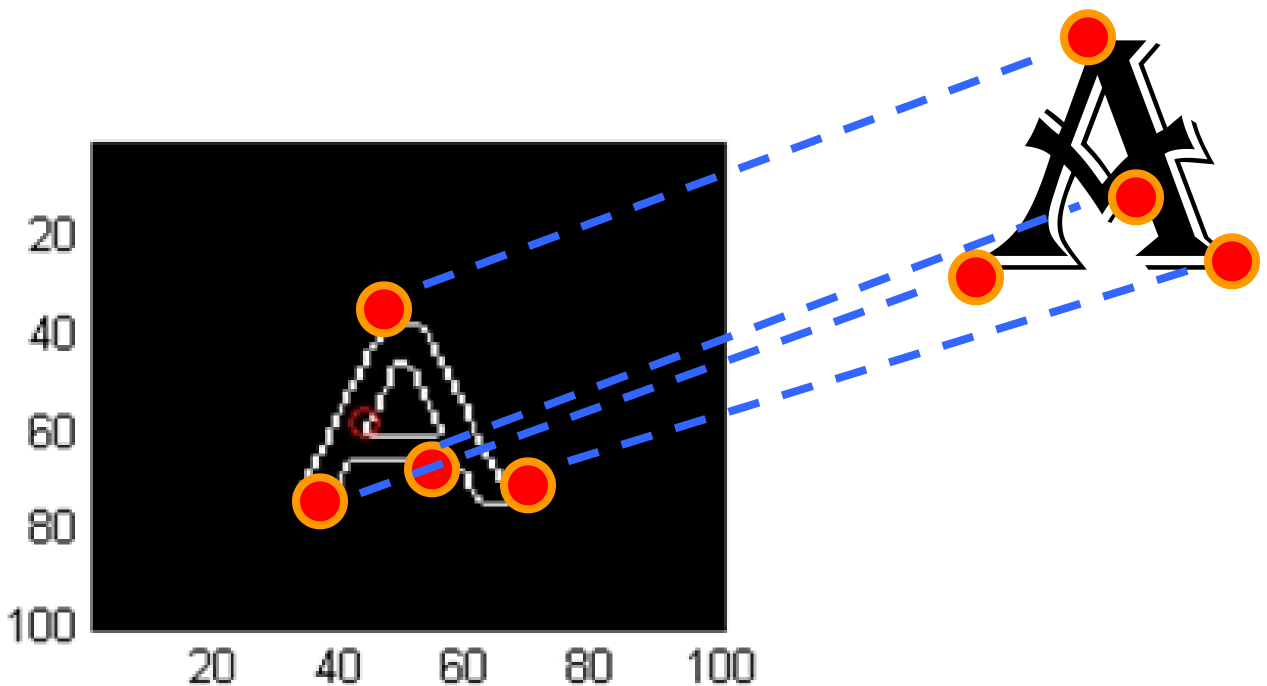
Critical issues: intra-class variability

"All models are wrong, but some are useful." Box and Draper 1979



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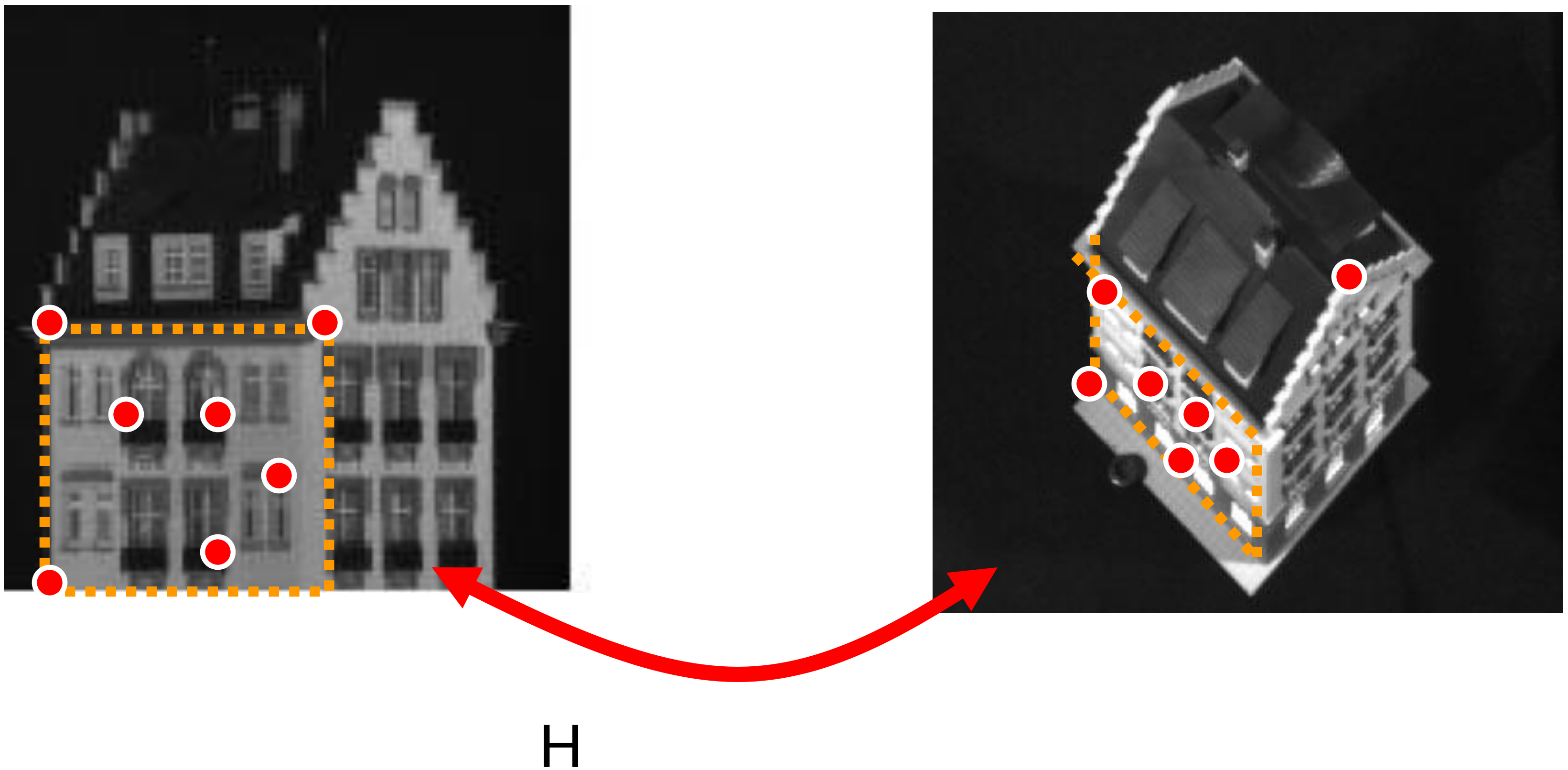
Critical issues: outliers



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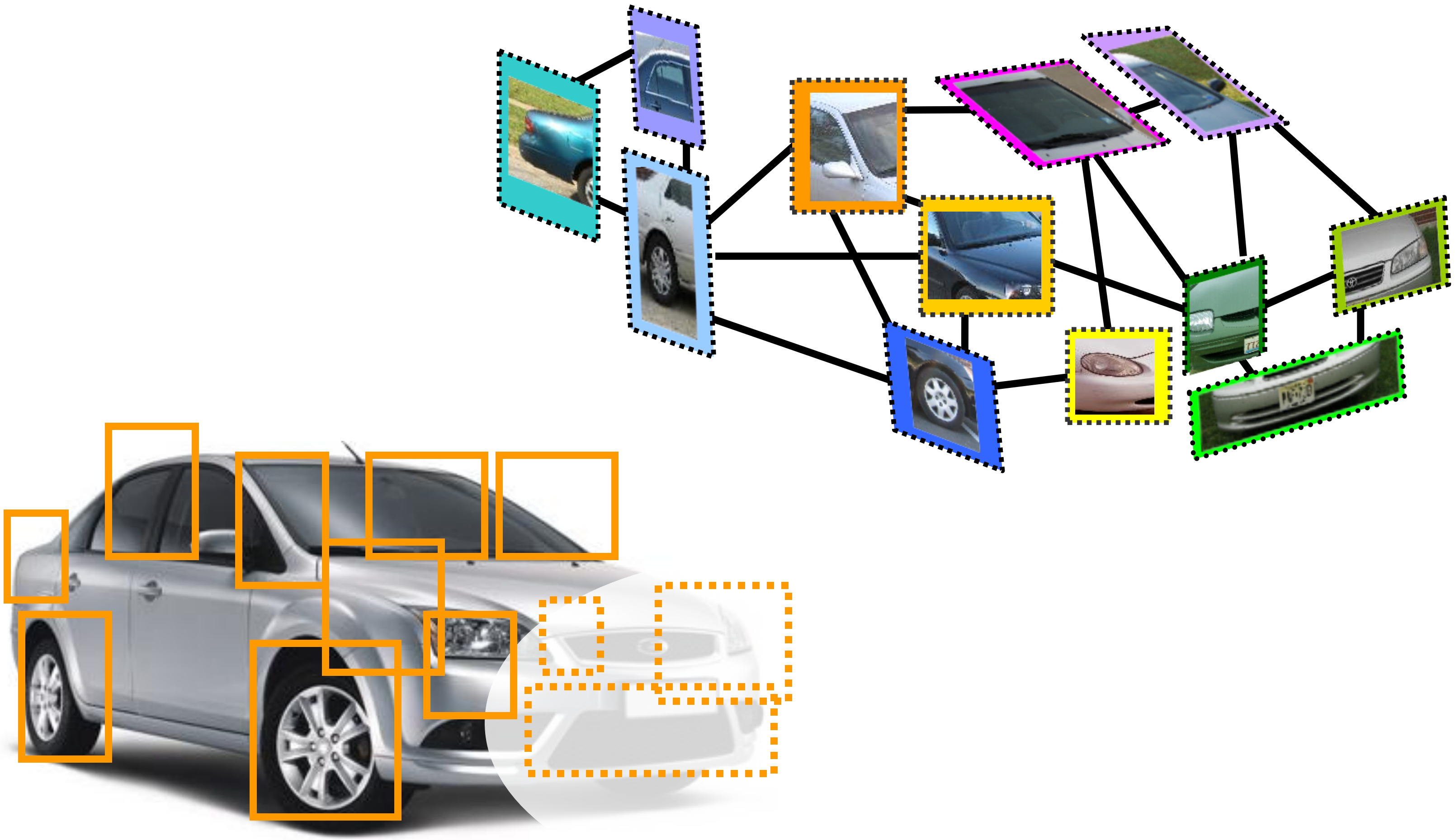
Critical issues: missing data (occlusions)



Slide from Silvio Savarese

Fitting and Alignment

- Design challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly

Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)

- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Slide from Derek Hoiem

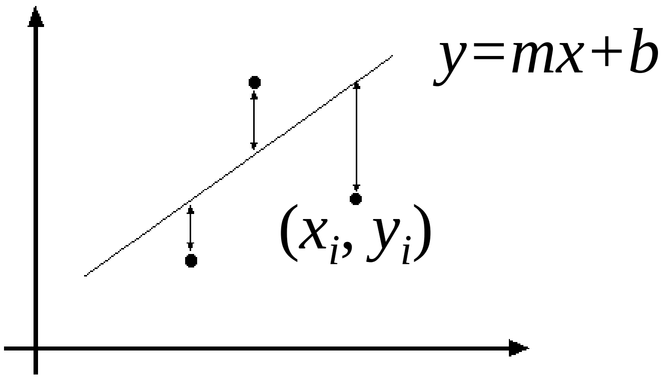
Simple example: Fitting a line

Slide from Derek Hoiem

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m\,x_i + b$
- Find (m, b) to minimize

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$



$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \|\mathbf{Ap} - \mathbf{y}\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

$$\frac{dE}{dB} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

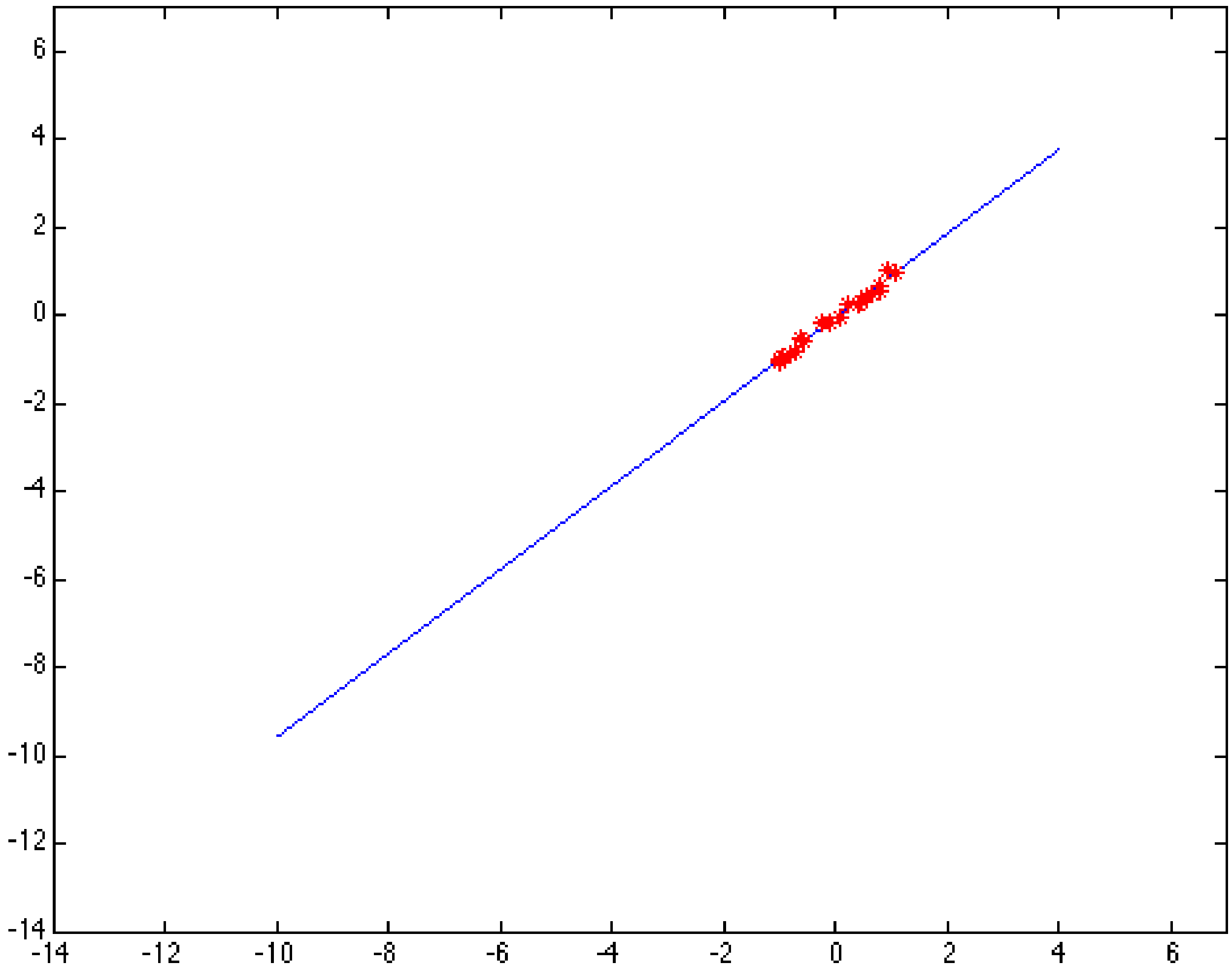
$$\boxed{\text{Matlab: } \texttt{p = A \textbackslash\ y;}}$$

$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{y}$$

Modified from S. Lazebnik

Least squares: Robustness to noise

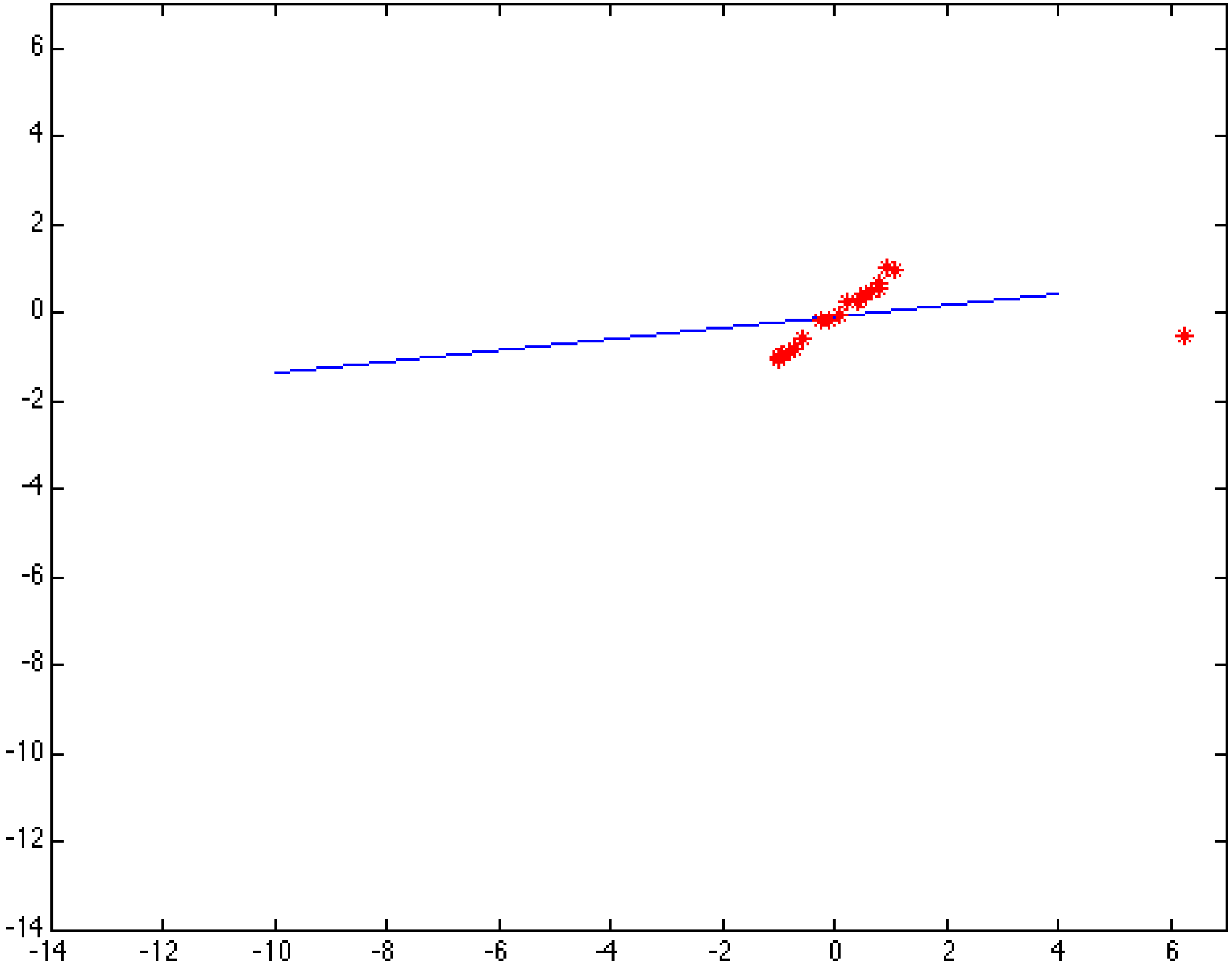
Least squares fit to the red points:



Slides from Svetlana Lazebnik

Least squares: Robustness to noise

Least squares fit with an outlier:



Problem: squared error heavily penalizes outliers

Search / Least squares conclusions

Good

- Clearly specified objective
- Optimization is easy (for least squares)

Bad

- Not appropriate for non-convex objectives
  - May get stuck in local minima
- Sensitive to outliers
  - Bad matches, extra points
- Doesn’t allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

Slide from Derek Hoiem

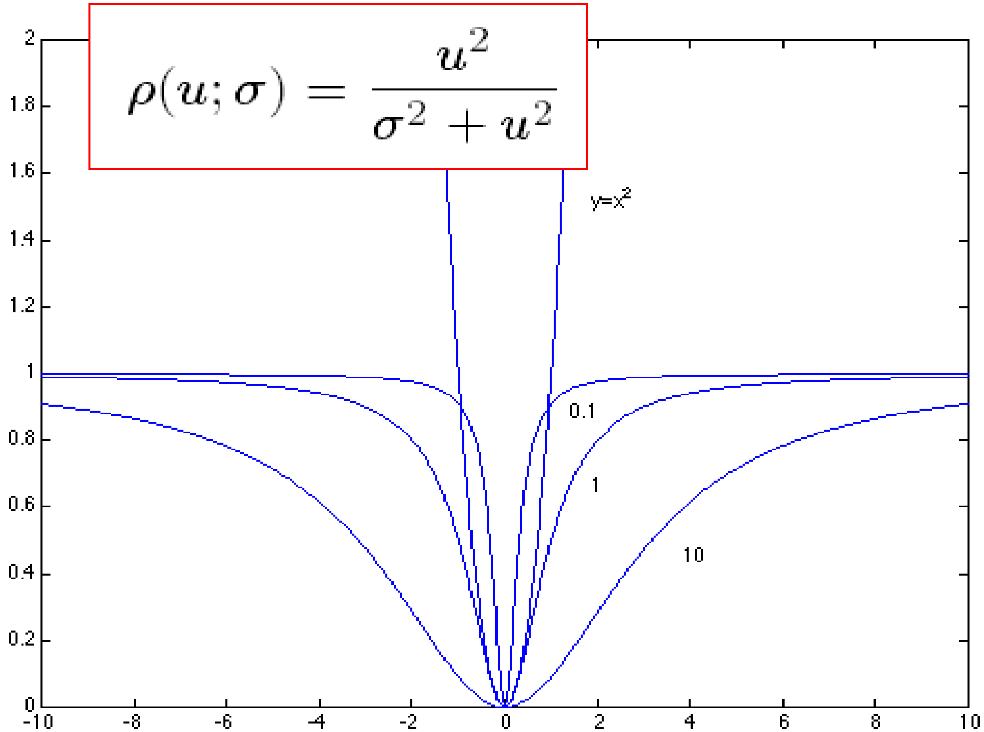
Robust least squares (to deal with outliers)

General approach:

minimize $$\sum_i \rho\left(u_i\left(x_i, \theta\right); \sigma\right) \qquad u = \sum_{i=1}^{n} (y_i - m x_i - b)^2$$

$u_i(x_i, \theta)$ – residual of i^th point w.r.t. model parameters ϑ
ρ – robust function with scale parameter σ

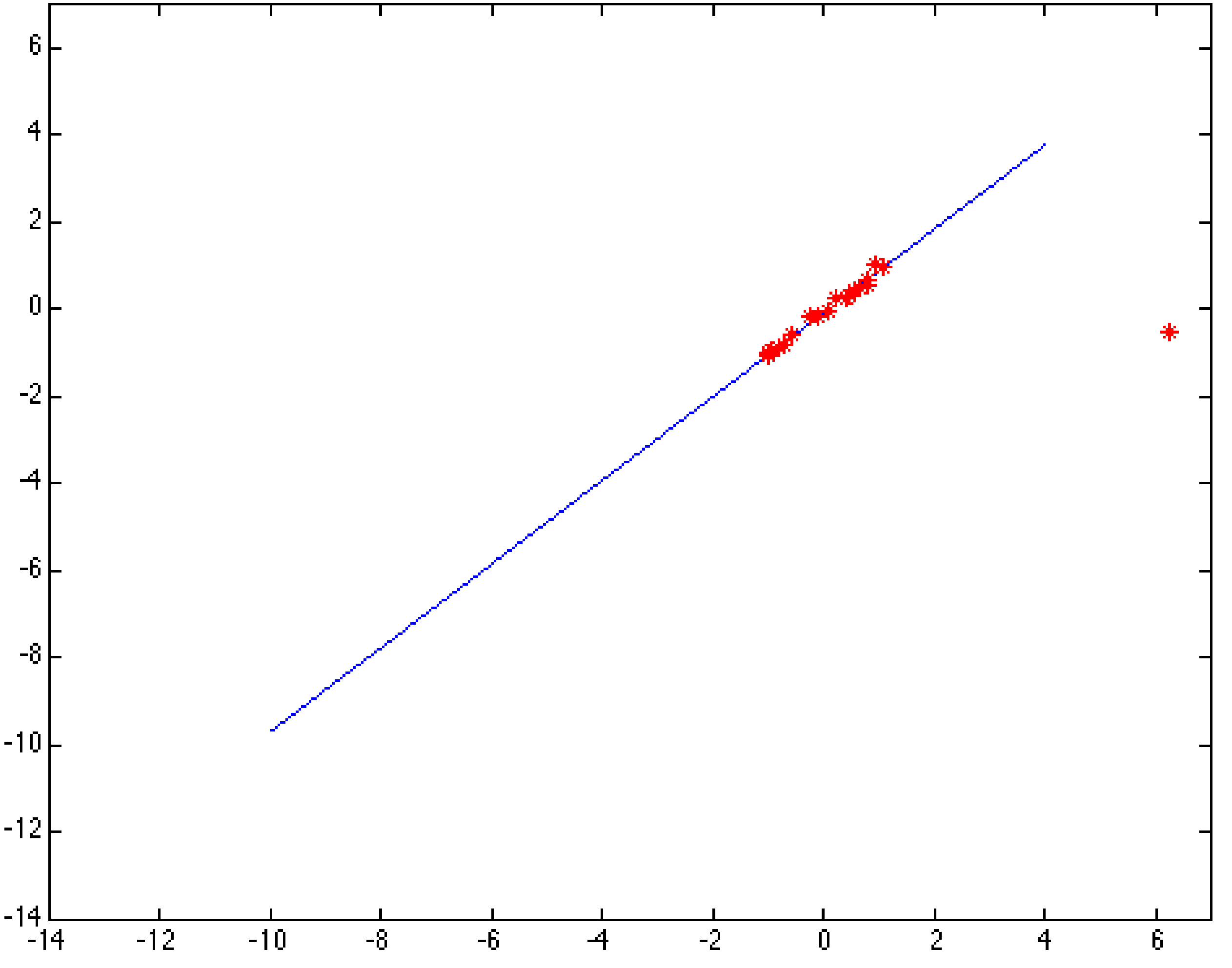
$$\rho(u;\sigma) = \frac{u^2}{\sigma^2 + u^2}$$



The robust function ρ

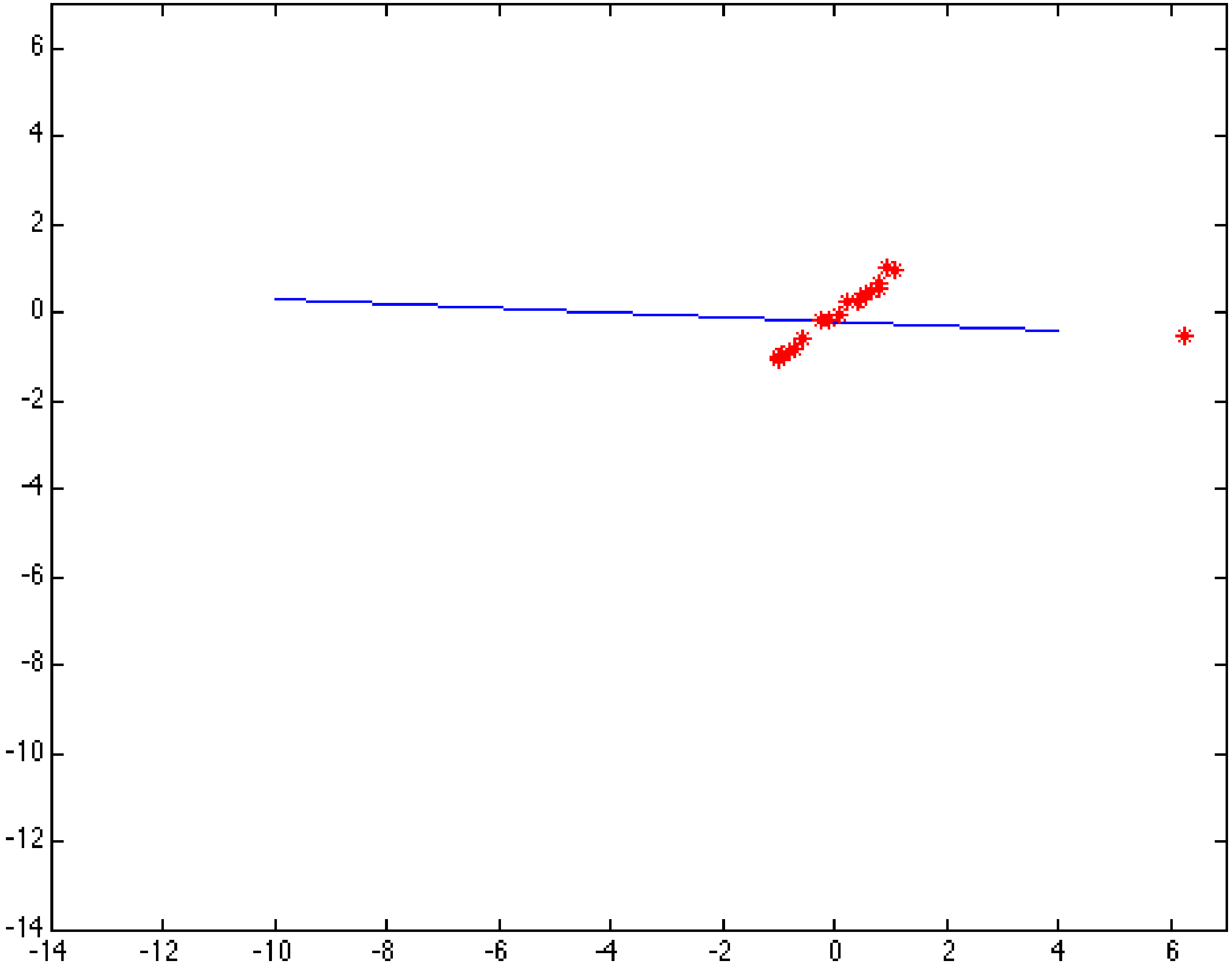
- Favors a configuration with small residuals
- Constant penalty for large residuals

Choosing the scale: Just right



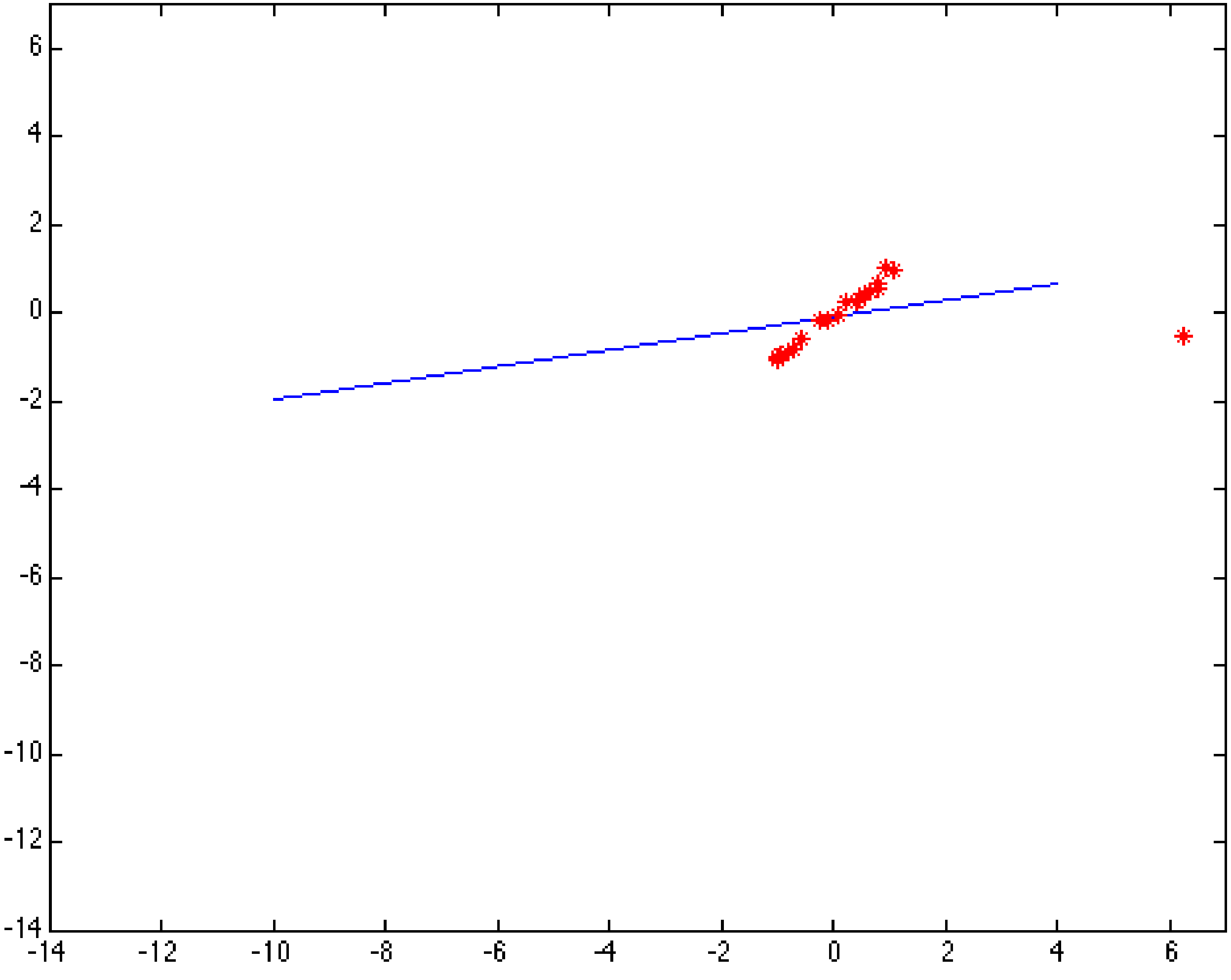
The effect of the outlier is minimized

Choosing the scale: Too small



The error value is almost the same for every point and the fit is very poor

Choosing the scale: Too large



Behaves much the same as least squares

Robust estimation: Details

- Robust fitting is a nonlinear optimization problem that must be solved iteratively
- Least squares solution can be used for initialization
- Adaptive choice of scale: approx. 1.5 times median residual (F&P, Sec. 15.5.1)

Hypothesize and test

1. Propose parameters
   - Try all possible
   - Each point votes for all consistent parameters
   - Repeatedly sample enough points to solve for parameters

2. Score the given parameters
   - Number of consistent points, possibly weighted by distance

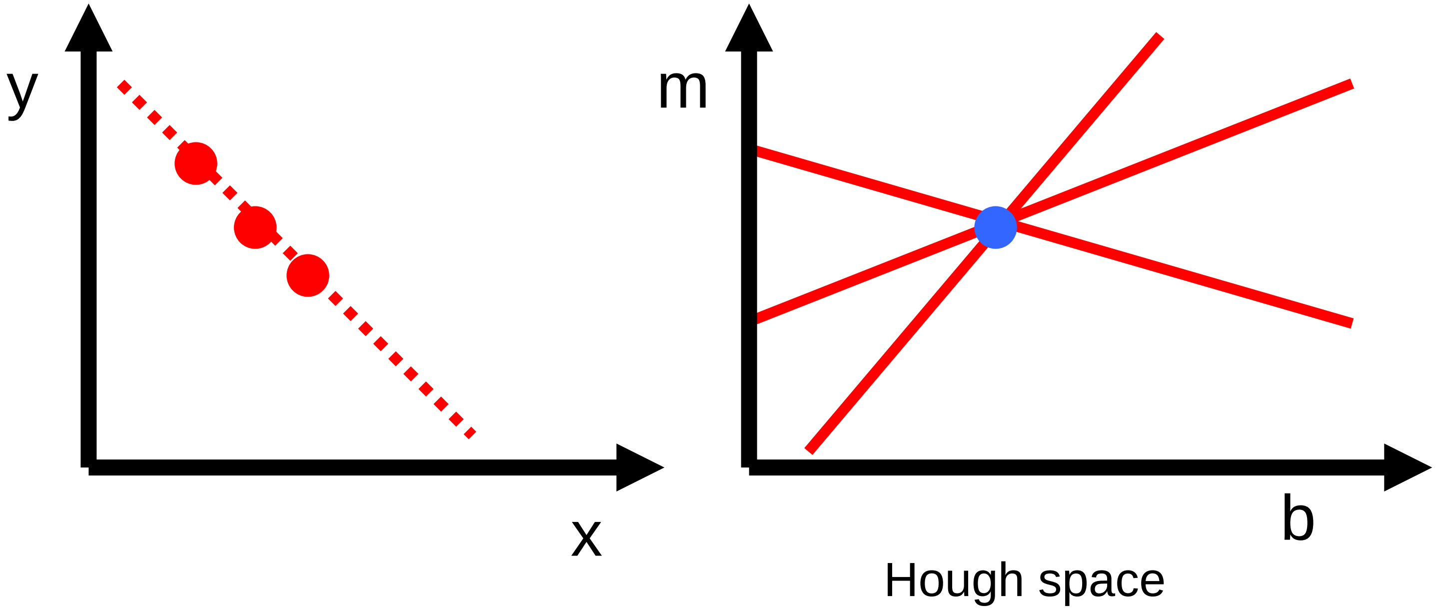
3. Choose from among the set of parameters
   - Global or local maximum of scores

4. Possibly refine parameters using inliers

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

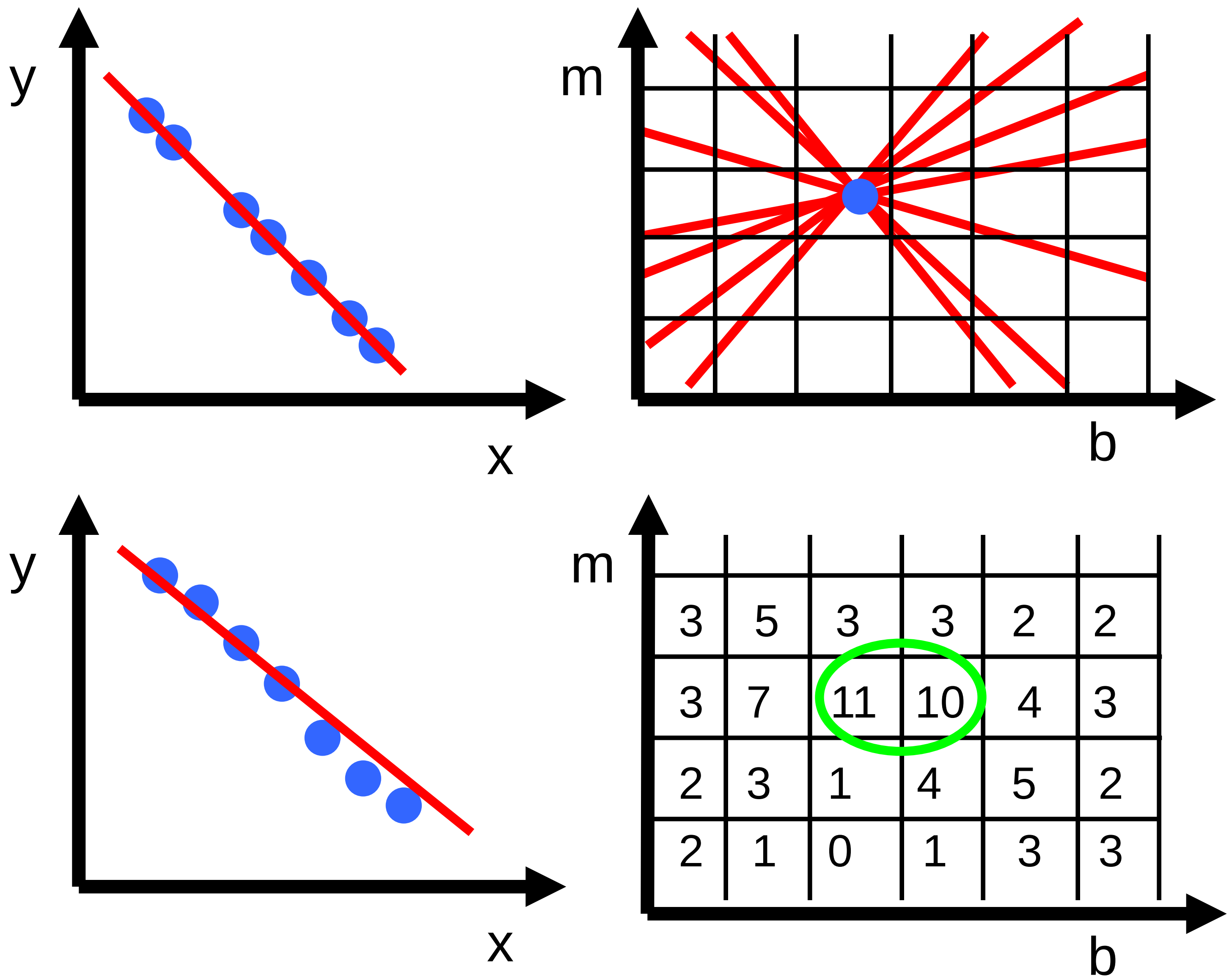
Given a set of points, find the curve or line that explains the data points best



$$y = m\,x + b$$

Slide from S. Savarese

Hough transform



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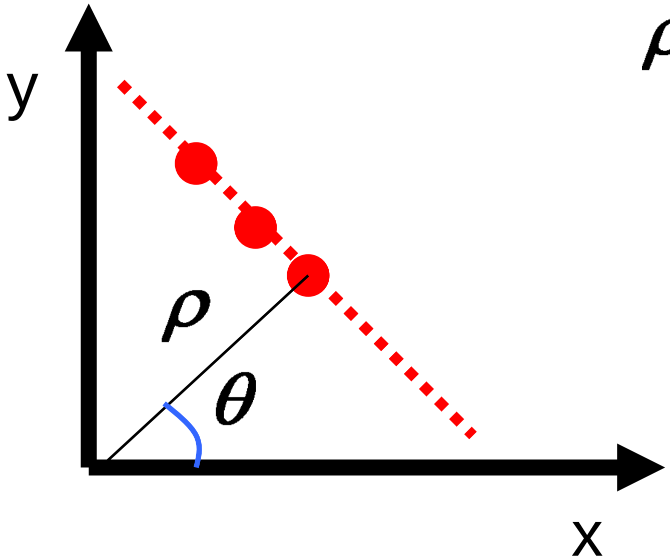
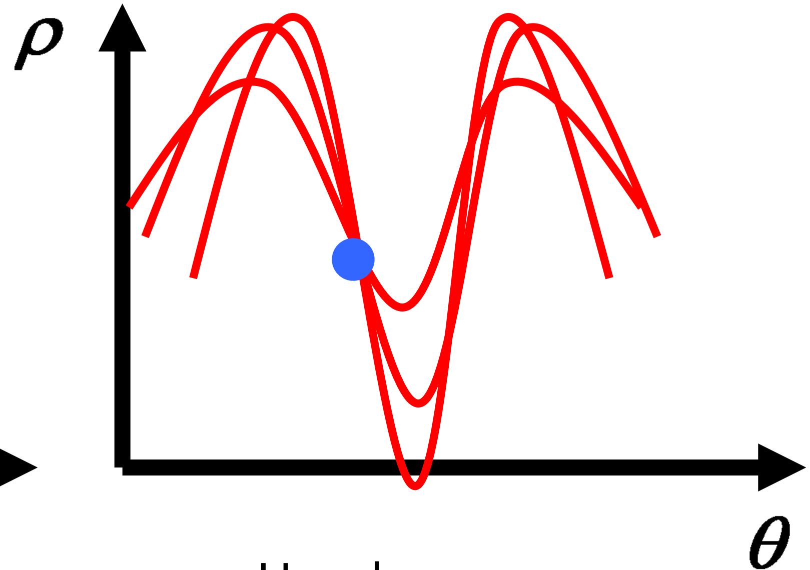
Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Issue : parameter space [m,b] is unbounded…

Use a polar representation for the parameter space





$$x\cos\theta + y\sin\theta = \rho$$

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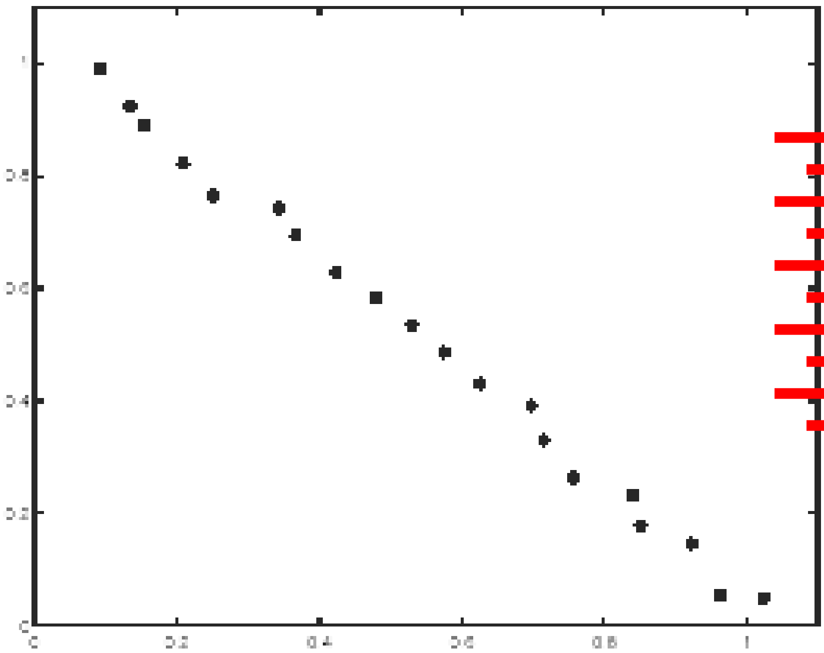
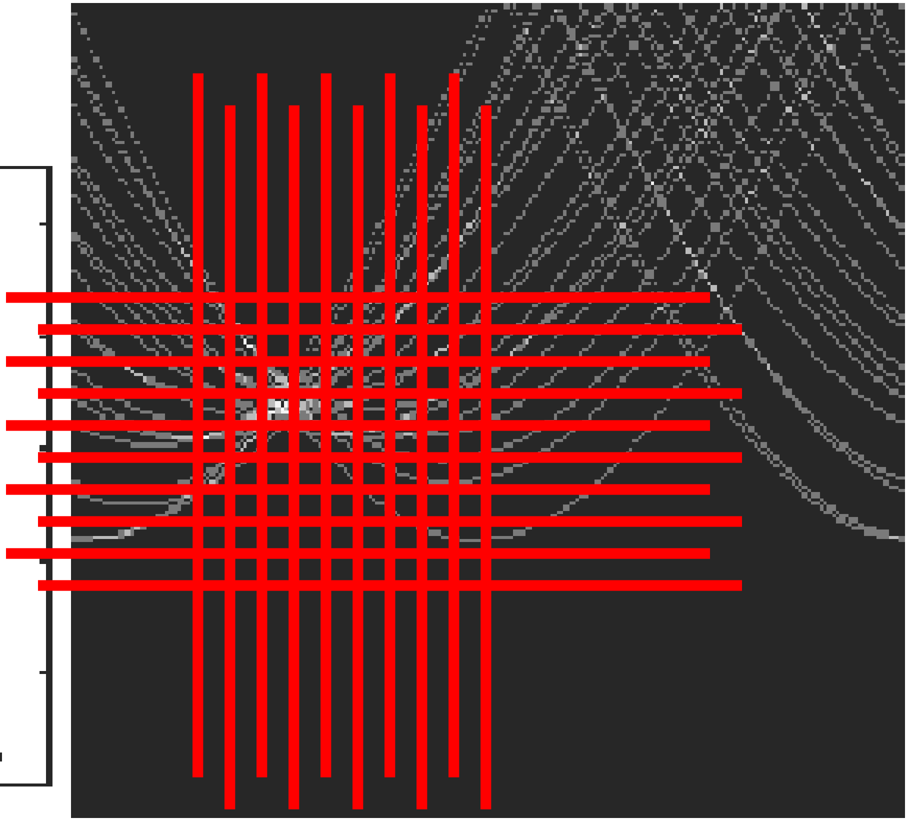
Hough transform - experiments

Noisy data



features



votes

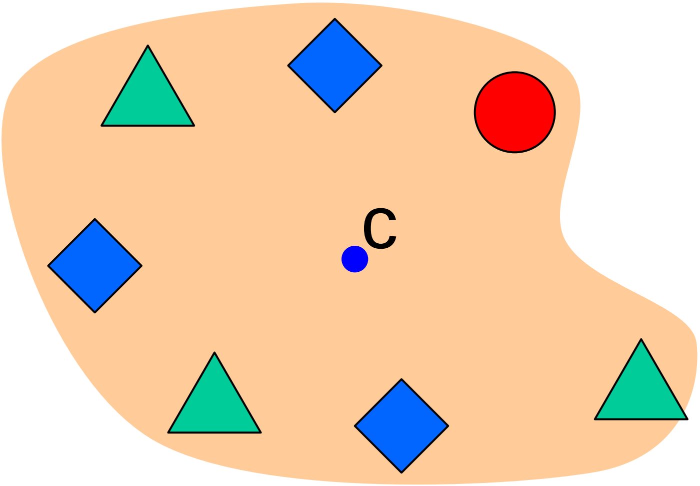
Issue: Grid size needs to be adjusted…

Slide from S. Savarese

Generalized Hough transform

- We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration

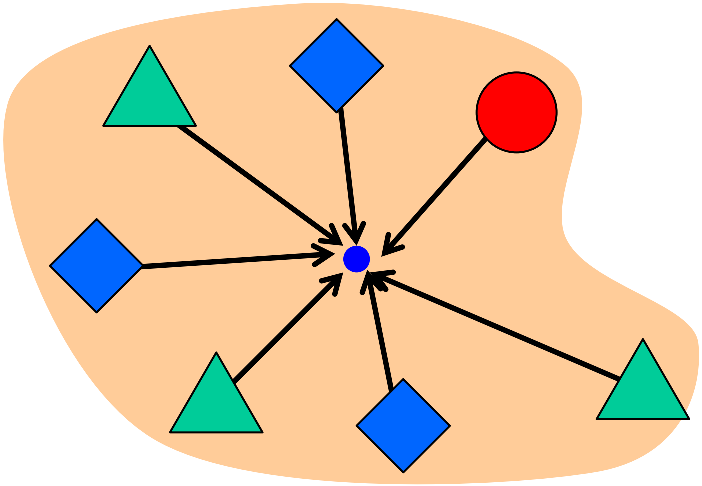
Template



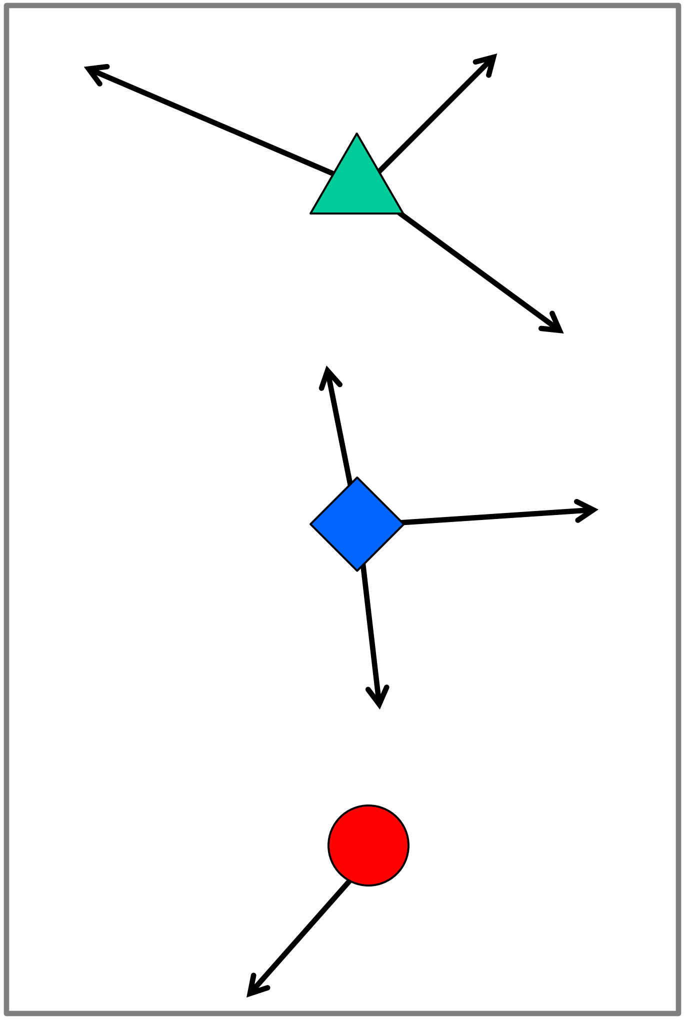
Generalized Hough transform

- Template representation: for each type of landmark point, store all possible displacement vectors towards the center

Template



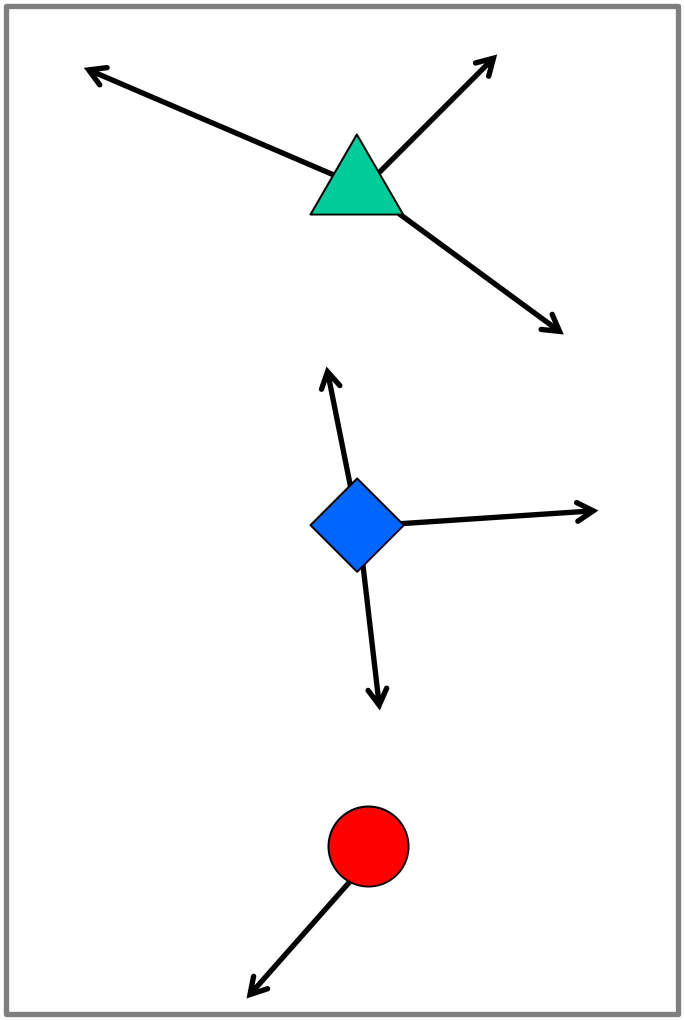
Model



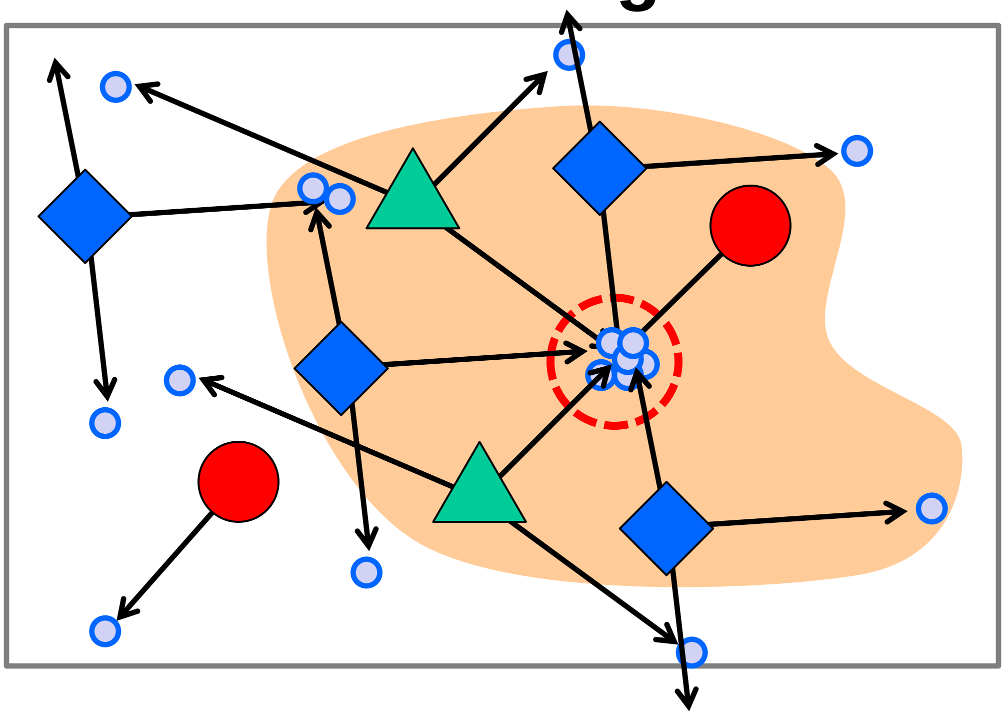
Generalized Hough transform

- Detecting the template:
  - For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model

Model

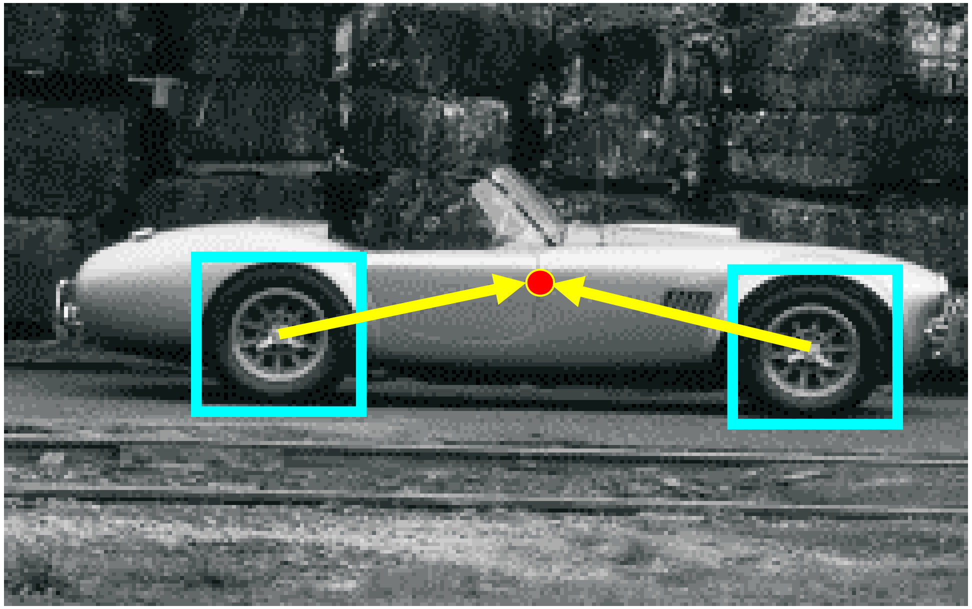


Test image



Application in recognition

- Index displacements by “visual codeword”



training image

visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Application in recognition

- Index displacements by "visual codeword"

test image

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](), ECCV Workshop on Statistical Learning in Computer Vision 2004

Hough transform conclusions

Good

- Robust to outliers: each point votes separately
- Fairly efficient (often faster than trying all sets of parameters)
- Provides multiple good fits

Bad

- Some sensitivity to noise
- Bin size trades off between noise tolerance, precision, and speed/memory
  - Can be hard to find sweet spot
- Not suitable for more than a few parameters
  - grid size grows exponentially

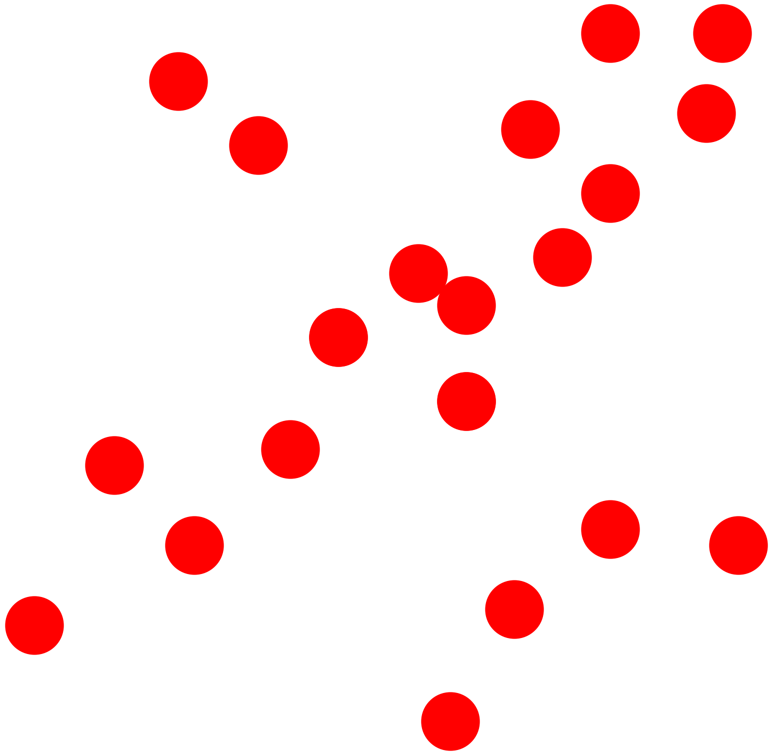
Common applications

- Line fitting (also circles, ellipses, etc.)
- Object instance recognition (parameters are affine transform)
- Object category recognition (parameters are position/scale)

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.



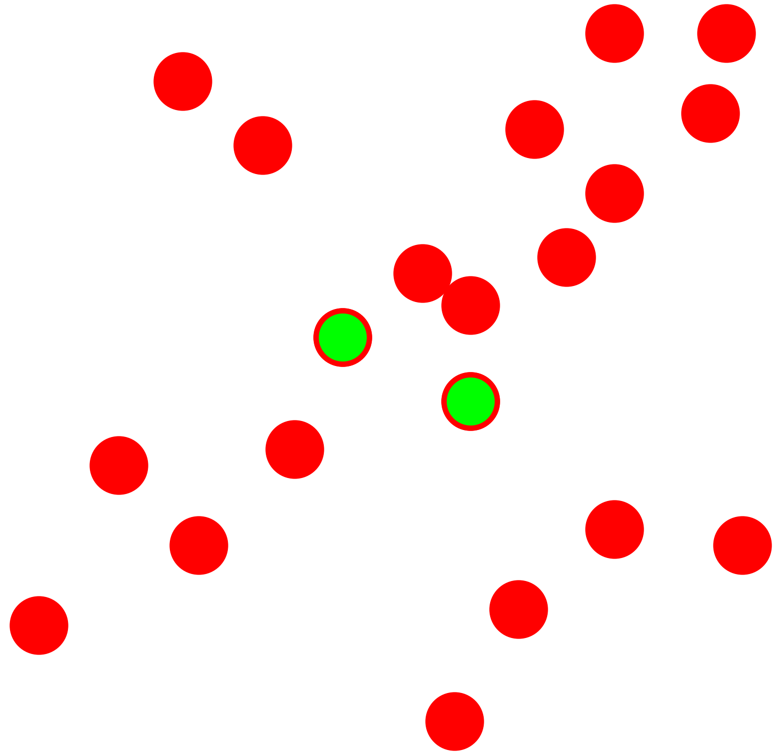
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



Algorithm:

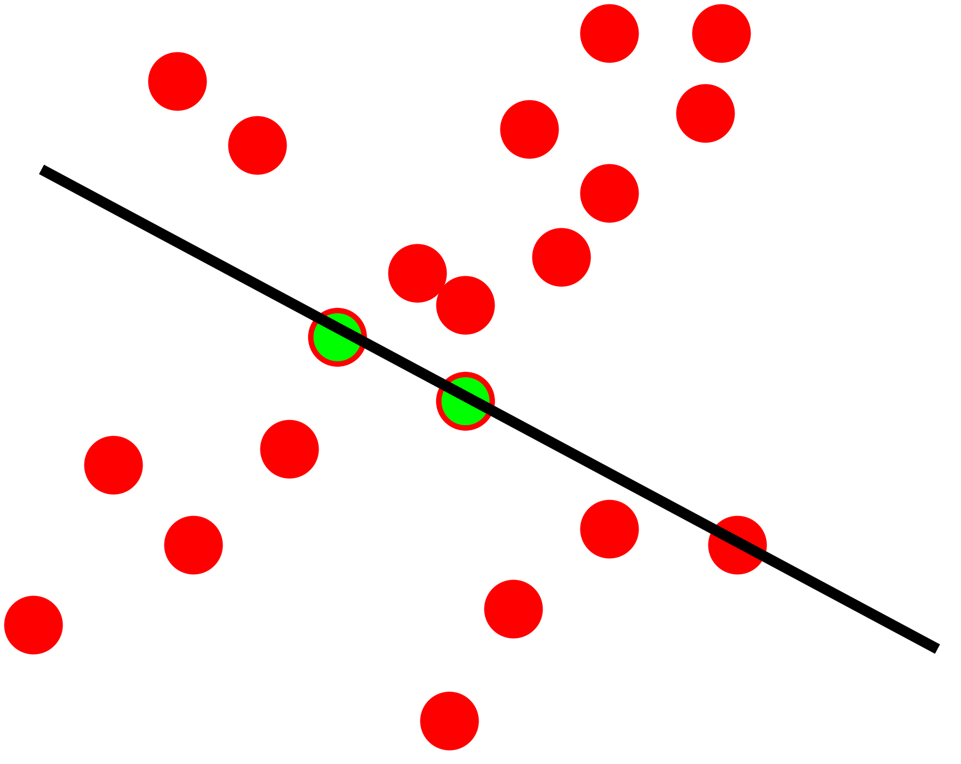
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



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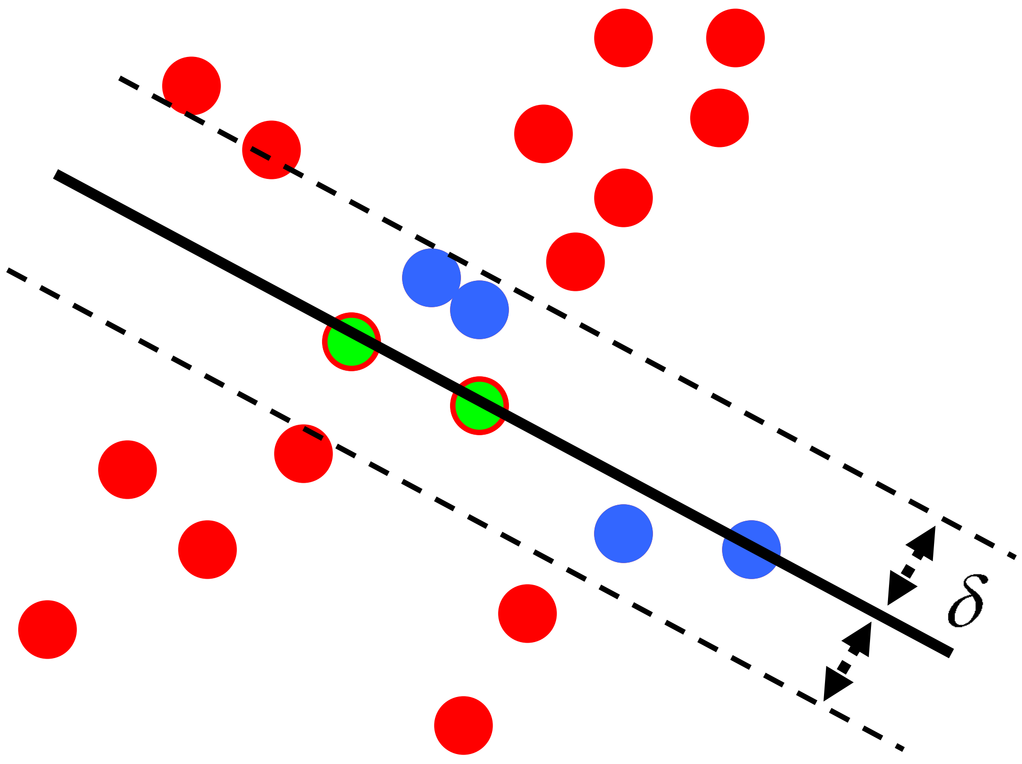
RANSAC

Line fitting example



$$N_I = 6$$

Algorithm:

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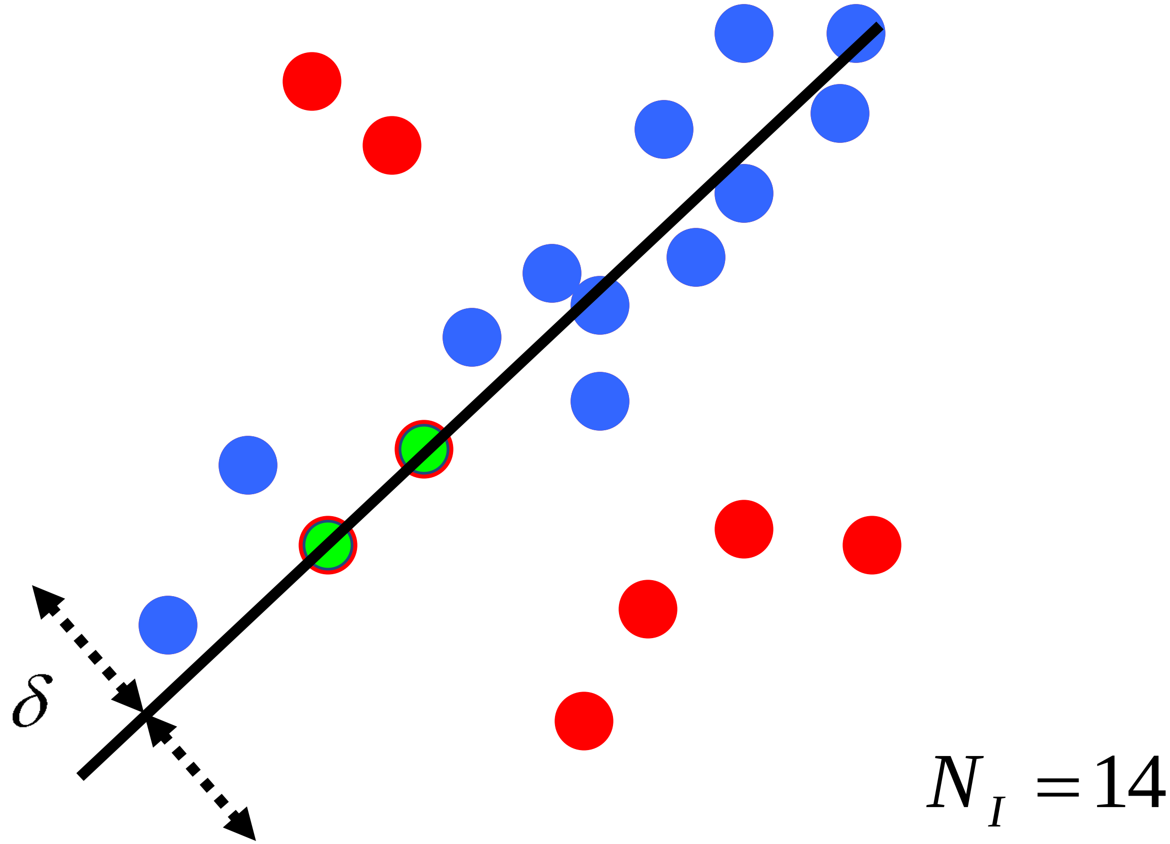
RANSAC



Algorithm:

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2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Choosing the parameters

- Initial number of points *s*
  - Typically minimum number needed to fit the model
- Distance threshold *t*
  - Choose *t* so probability for inlier is *p* (e.g. 0.95)
  - Zero-mean Gaussian noise with std. dev. σ: $t^2=3.84\sigma^2$
- Number of samples *N*
  - Choose *N* so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)

$$\boxed{\left(1-\left(1-e\right)^s\right)^N = 1-p}$$

$$N = \log\left(1-p\right)/\log\left(1-\left(1-e\right)^s\right)$$

	proportion of outliers e						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Source: M. Pollefeys

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of parameters than Hough transform
- Parameters are easier to choose than Hough transform

Bad

- Computational time grows quickly with fraction of outliers and number of parameters
- Not good for getting multiple fits

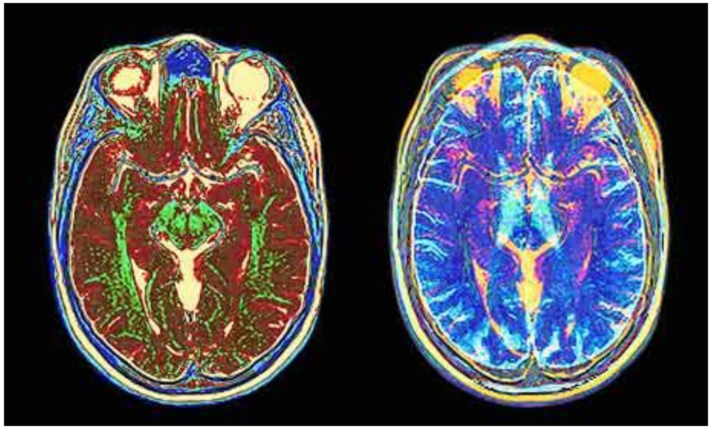
Common applications

- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)

What if you want to align but have no prior matched pairs?

- Hough transform and RANSAC not applicable

- Important applications



Medical imaging: match brain scans or contours

Robotics: match point clouds

Slide from Derek Hoiem

Iterative Closest Points (ICP) Algorithm

Goal: estimate transform between two dense sets of points

1. **Assign** each point in {Set 1} to its nearest neighbor in {Set 2}
2. **Estimate** transformation parameters
   – e.g., least squares or robust least squares
3. **Transform** the points in {Set 1} using estimated parameters
4. **Repeat** steps 1-3 until change is very small

Slide from Derek Hoiem