Machine Learning: Classification

Computer Vision
CS 143, Brown

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The machine learning framework

- Apply a prediction function to a feature representation of the image to get the desired output:

\[
\begin{align*}
f(\text{apple}) &= \text{“apple”} \\
f(\text{tomato}) &= \text{“tomato”} \\
f(\text{cow}) &= \text{“cow”}
\end{align*}
\]
The machine learning framework

\[ y = f(x) \]

- **Training**: given a *training set* of labeled examples \( \{(x_1, y_1), \ldots, (x_N, y_N)\} \), estimate the prediction function \( f \) by minimizing the prediction error on the training set.
- **Testing**: apply \( f \) to a never before seen *test example* \( x \) and output the predicted value \( y = f(x) \)

Slide credit: L. Lazebnik
**Steps**

**Training**
- Training Images
- Training Labels
- Image Features
- Learning model

**Testing**
- Test Image
- Image Features
- Learned model
- Prediction

Slide credit: D. Hoiem and L. Lazebnik
Features

• Raw pixels

• Histograms

• GIST descriptors

• ...
Classifiers: Nearest neighbor

\[ f(x) = \text{label of the training example nearest to } x \]

- All we need is a distance function for our inputs
- No training required!
Classifiers: Linear

- Find a linear function to separate the classes:

\[ f(x) = \text{sgn}(w \cdot x + b) \]
Many classifiers to choose from

- SVM
- Neural networks
- Naïve Bayes
- Bayesian network
- Logistic regression
- Randomized Forests
- Boosted Decision Trees
- K-nearest neighbor
- RBMs
- Etc.

Which is the best one?
Recognition task and supervision

- Images in the training set must be annotated with the “correct answer” that the model is expected to produce.

Contains a motorbike
Spectrum of supervision

- Unsupervised
- “Weakly” supervised
- Fully supervised

Definition depends on task

Slide credit: L. Lazebnik
Generalization

- How well does a learned model generalize from the data it was trained on to a new test set?

Training set (labels known)  Test set (labels unknown)

Slide credit: L. Lazebnik
Generalization

• Components of generalization error
  – **Bias**: how much the average model over all training sets differ from the true model?
    • Error due to inaccurate assumptions/simplifications made by the model
  – **Variance**: how much models estimated from different training sets differ from each other

• **Underfitting**: model is too “simple” to represent all the relevant class characteristics
  – High bias and low variance
  – High training error and high test error

• **Overfitting**: model is too “complex” and fits irrelevant characteristics (noise) in the data
  – Low bias and high variance
  – Low training error and high test error

Slide credit: L. Lazebnik
Bias-Variance Trade-off

- Models with too few parameters are inaccurate because of a large bias (not enough flexibility).

- Models with too many parameters are inaccurate because of a large variance (too much sensitivity to the sample).
Bias-Variance Trade-off

\[ E(MSE) = \text{noise}^2 + \text{bias}^2 + \text{variance} \]

Unavoidable error

Error due to incorrect assumptions

Error due to variance of training samples

See the following for explanations of bias-variance (also Bishop’s “Neural Networks” book):

- [http://www.inf.ed.ac.uk/teaching/courses/mlsc/Notes/Lecture4/BiasVariance.pdf](http://www.inf.ed.ac.uk/teaching/courses/mlsc/Notes/Lecture4/BiasVariance.pdf)
Bias-variance tradeoff

Underfitting  Overfitting

High Bias  Low Bias  High Variance  Low Variance

Complexity  Error

Slide credit: D. Hoiem
Bias-variance tradeoff

![Graph showing the relationship between test error, complexity, bias, and variance.](slide-credit: D. Hoiem)
Effect of Training Size

Fixed prediction model

Error

Generalization Error

Number of Training Examples

Slide credit: D. Hoiem
Remember...

• No classifier is inherently better than any other: you need to make assumptions to generalize

• Three kinds of error
  – Inherent: unavoidable
  – Bias: due to over-simplifications
  – Variance: due to inability to perfectly estimate parameters from limited data
How to reduce variance?

• Choose a simpler classifier

• Regularize the parameters

• Get more training data
Very brief tour of some classifiers

- SVM
- Neural networks
- Naïve Bayes
- Bayesian network
- Logistic regression
- Randomized Forests
- Boosted Decision Trees
- K-nearest neighbor
- RBMs
- Etc.
Generative vs. Discriminative Classifiers

Generative Models
- Represent both the data and the labels
- Often, makes use of conditional independence and priors
- Examples
  - Naïve Bayes classifier
  - Bayesian network
- Models of data may apply to future prediction problems

Discriminative Models
- Learn to directly predict the labels from the data
- Often, assume a simple boundary (e.g., linear)
- Examples
  - Logistic regression
  - SVM
  - Boosted decision trees
- Often easier to predict a label from the data than to model the data

Slide credit: D. Hoiem
Classification

- Assign input vector to one of two or more classes
- Any decision rule divides input space into *decision regions* separated by *decision boundaries*
Nearest Neighbor Classifier

- Assign label of nearest training data point to each test data point

Voronoi partitioning of feature space for two-category 2D and 3D data

Source: D. Lowe
K-nearest neighbor

![Diagram of K-nearest neighbor visualization with points labeled as 'x' and 'o'.]
1-nearest neighbor
3-nearest neighbor
5-nearest neighbor
Using K-NN

• Simple, a good one to try first

• With infinite examples, 1-NN provably has error that is at most twice Bayes optimal error
Naïve Bayes
Using Naïve Bayes

- Simple thing to try for categorical data
- Very fast to train/test
Classifiers: Linear SVM
Classifiers: Linear SVM
Classifiers: Linear SVM
Datasets that are linearly separable work out great:

But what if the dataset is just too hard?

We can map it to a higher-dimensional space:
Nonlinear SVMs

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable:

\[ \Phi: \mathbf{x} \rightarrow \varphi(\mathbf{x}) \]
Nonlinear SVMs

• *The kernel trick*: instead of explicitly computing the lifting transformation \( \varphi(x) \), define a kernel function \( K \) such that

\[
K(x_i, x_j) = \varphi(x_i) \cdot \varphi(x_j)
\]

(to be valid, the kernel function must satisfy *Mercer’s condition*)

• This gives a nonlinear decision boundary in the original feature space:

\[
\sum_i \alpha_i y_i \varphi(x_i) \cdot \varphi(x) + b = \sum_i \alpha_i y_i K(x_i, x) + b
\]

Nonlinear kernel: Example

- Consider the mapping \( \varphi(x) = (x, x^2) \)

\[
\varphi(x) \cdot \varphi(y) = (x, x^2) \cdot (y, y^2) = xy + x^2 y^2
\]

\[
K(x, y) = xy + x^2 y^2
\]
Kernels for bags of features

• Histogram intersection kernel:

\[
I(h_1, h_2) = \sum_{i=1}^{N} \min(h_1(i), h_2(i))
\]

• Generalized Gaussian kernel:

\[
K(h_1, h_2) = \exp\left(-\frac{1}{A} D(h_1, h_2)^2\right)
\]

• \(D\) can be L1 distance, Euclidean distance, \(\chi^2\) distance, etc.

Summary: SVMs for image classification

1. Pick an image representation (in our case, bag of features)
2. Pick a kernel function for that representation
3. Compute the matrix of kernel values between every pair of training examples
4. Feed the kernel matrix into your favorite SVM solver to obtain support vectors and weights
5. At test time: compute kernel values for your test example and each support vector, and combine them with the learned weights to get the value of the decision function

Slide credit: L. Lazebnik
What about multi-class SVMs?

- Unfortunately, there is no “definitive” multi-class SVM formulation
- In practice, we have to obtain a multi-class SVM by combining multiple two-class SVMs

- One vs. others
  - Training: learn an SVM for each class vs. the others
  - Testing: apply each SVM to test example and assign to it the class of the SVM that returns the highest decision value

- One vs. one
  - Training: learn an SVM for each pair of classes
  - Testing: each learned SVM “votes” for a class to assign to the test example

Slide credit: L. Lazebnik
SVMs: Pros and cons

• Pros
  • Many publicly available SVM packages: http://www.kernel-machines.org/software
  • Kernel-based framework is very powerful, flexible
  • SVMs work very well in practice, even with very small training sample sizes

• Cons
  • No “direct” multi-class SVM, must combine two-class SVMs
  • Computation, memory
    – During training time, must compute matrix of kernel values for every pair of examples
    – Learning can take a very long time for large-scale problems
Classifiers: Decision Trees
Ensemble Methods: Boosting

Discrete AdaBoost (Freund & Schapire 1996b)

1. Start with weights $w_i = 1/N, \ i = 1, \ldots, N$.

2. Repeat for $m = 1, 2, \ldots, M$:
   
   (a) Fit the classifier $f_m(x) \in \{-1, 1\}$ using weights $w_i$ on the training data.
   
   (b) Compute $\text{err}_m = E_w[1_{(y \neq f_m(x))}], \ \ c_m = \log((1 - \text{err}_m)/\text{err}_m)$.
   
   (c) Set $w_i \leftarrow w_i \exp[c_m \cdot 1_{(y \neq f_m(x_i))}], \ i = 1, 2, \ldots, N$, and renormalize so that $\sum_i w_i = 1$.

3. Output the classifier $\text{sign} \left[ \sum_{m=1}^{M} c_m f_m(x) \right]$
Boosted Decision Trees

\[ P(label \mid \text{good segment}, \text{data}) \]

[Collins et al. 2002]
Using Boosted Decision Trees

• Flexible: can deal with both continuous and categorical variables

• How to control bias/variance trade-off
  – Size of trees
  – Number of trees

• Boosting trees often works best with a small number of well-designed features

• Boosting “stubs” can give a fast classifier
Two ways to think about classifiers

1. What is the objective? What are the parameters? How are the parameters learned? How is the learning regularized? How is inference performed?

2. How is the data modeled? How is similarity defined? What is the shape of the boundary?
## Comparison

### Learning Objective

<table>
<thead>
<tr>
<th>Method</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Naïve Bayes</strong></td>
<td>$\max \sum_i \left[ \sum_j \log P(\xi_{ij}</td>
</tr>
<tr>
<td><strong>Logistic Regression</strong></td>
<td>$\max \sum_i \log \left( P(\xi_i</td>
</tr>
<tr>
<td><strong>Linear SVM</strong></td>
<td>$\min \lambda \sum_i \xi_i + \frac{1}{2} | \theta |$ such that $y_i \theta^T x \geq 1 - \xi_i \ \forall i$</td>
</tr>
<tr>
<td><strong>Kernelized SVM</strong></td>
<td>complicated to write</td>
</tr>
<tr>
<td><strong>Nearest Neighbor</strong></td>
<td><strong>most similar features → same label</strong></td>
</tr>
</tbody>
</table>

### Training

<table>
<thead>
<tr>
<th>Method</th>
<th>Training Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Naïve Bayes</strong></td>
<td>$\theta_{kj} = \frac{\sum \delta(\xi_{ij} = 1 \wedge y_i = k \Rightarrow r)}{\sum \delta(\xi_i = k \Rightarrow Kr)}$</td>
</tr>
<tr>
<td><strong>Logistic Regression</strong></td>
<td>Gradient ascent $\theta^T x &gt; 0$</td>
</tr>
<tr>
<td><strong>Linear SVM</strong></td>
<td>Linear programming $\theta^T x &gt; 0$</td>
</tr>
<tr>
<td><strong>Kernelized SVM</strong></td>
<td>Quadratic programming $\sum_i y_i \alpha_i K(\xi_i, x) &gt; 0$</td>
</tr>
</tbody>
</table>

### Inference

<table>
<thead>
<tr>
<th>Method</th>
<th>Inference Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Naïve Bayes</strong></td>
<td>$\hat{x} = \arg\min_y P(y</td>
</tr>
<tr>
<td><strong>Logistic Regression</strong></td>
<td>$y$ reached where $i = \arg\min_i K(\xi_i, x)$</td>
</tr>
<tr>
<td><strong>Linear SVM</strong></td>
<td>$\theta^T x &gt; 0$</td>
</tr>
</tbody>
</table>

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*Slide credit: D. Hoiem*
What to remember about classifiers

• No free lunch: machine learning algorithms are tools, not dogmas

• Try simple classifiers first

• Better to have smart features and simple classifiers than simple features and smart classifiers

• Use increasingly powerful classifiers with more training data (bias-variance tradeoff)
Some Machine Learning References

• General
  – Christopher Bishop, *Neural Networks for Pattern Recognition*, Oxford University Press, 1995

• Adaboost

• SVMs