Adversarial Search

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“Chess is the Drosophila of Artificial Intelligence”
Kronrod, c. 1966

TuroChamp, 1948
Why Study Games?

Of interest:
• Many human activities (especially intellectual ones) can be modeled as games.
• Prestige.

Convenient:
• Perfect information.
• Concise, precise rules.
• Well defined “score”.
“Solved” Games

A game is solved if an optimal strategy is known.

Strong solved: all positions.
Weakly solved: some (start) positions.
Typical Game Setting

Games are usually:
- 2 player
- Alternating
- Zero-sum
  - Gain for one loss for another.
- Perfect information

Very much like search:
- Start state
- Successor function
- Terminal states (many)
- Objective function

but alternating control.
Game Trees

player 1 moves

player 2 moves

player 1 moves
Key Differences vs. Search

you select to max score

they select to min score

only get score here
Minimax Algorithm

Max player: select action to maximize return.
Min player: select action to minimize return.

This is optimal for both players (if zero sum). Assumes perfect play, worst case.

Can run as depth first:
• Time $O(b^d)$
• Space $O(bd)$

Require the agent to evaluate the whole tree.
Minimax
In Practice

*Depth is too deep.*
  - 10s to 100s of moves.

*Breadth is too broad.*
  - Chess: 35, Go: 361.

*Full search never terminates for non-trivial games.*
What Is To Be Done?

Must terminate the search early.
Solution: substitute evaluation function.

- Like a heuristic - estimate value.
- In this case, probability of win.

- Common strategy:
  - Run to fixed depth then estimate.
  - Careful lookahead to depth $d$, then guess.
Search Control

Horizon Effects
• What if something interesting at horizon + 1?
• How do you know?

More sophisticated strategies:
• When to generate more nodes?
• How to selectively expand the frontier?
• How to allocate fixed move time?
Alpha-Beta

max

min
Alpha Beta Pruning

Single most useful search control method:
  • Throw away whole branches.
  • Use the min-max behavior.

  • Cutoff search at min nodes where max can force a better outcome.

  • Cutoff search at max nodes when min can force a worse outcome.

Resulting algorithm: alpha-beta pruning.
Alpha-Beta

Empirically: *square roots* branching factor.
  • Effectively doubles the search horizon.

Alpha-beta makes the difference between novice and expert computer game players. *Most successful players use* alpha-beta.
Deep Blue (1997)

480 Special Purpose Chips
200 million positions/sec
Search depth 6-8 moves (up to 20)
Games of Chance

What if there is a chance element?
Stochasticity

An outcome is called *stochastic* when it is determined at random.

![Dice with probabilities](image)

The probabilities for each outcome sum to 1: $p=1/6$. 

1. $p=1/6$
2. $p=1/6$
3. $p=1/6$
4. $p=1/6$
5. $p=1/6$
6. $p=1/6$
Stochasticity

How to factor in stochasticity?

Agent does not get to *choose*.
  • Selecting the *max* outcome is optimistic.
  • Selecting the *min* outcome is pessimistic.

Must be probability-aware.
Expectation

What is the average die value?

\[
\frac{(1 + 2 + 3 + 4 + 5 + 6)}{6} = 3.5
\]

This factors in both probabilities and the value of event.

In general, given random event \( x \) and function \( f(x) \):

\[
E[f(x)] = \sum_{x} P(x) f(x)
\]

Insert expectation layer to accommodate stochastic events.
ExpectiMax

stochastic (expectation)

you select (max)

they select to min score

stochastic (expectation)
Games Today

World champion level:
• Backgammon
• Chess
• Checkers (solved)
• Othello
• Some poker types:

Perform well:
• Bridge
• Other poker types

Far off: Go
Go
Very Recently

Lee Sedol

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AlphaGo
(Google Deepmind)