Introduction

The hyper-intelligent pan-dimensional beings have decided to visit the legendary planet Magrathea, which is known for selling luxury planets. However, due to internet connectivity issues, the aliens can not use Google Maps to get directions to the planet and need to use an old atlas to navigate themselves. In order to set the correct coordinates, the aliens need to read and understand the map symbols, but the atlas is so old that none of the aliens remember how to understand and reason about the map’s directions. In this assignment, you will help the aliens implement knowledge representation and reasoning so that they can understand their map to get to Magrathea.

Question 1

Show that both of De Morgan’s Laws are true, using truth tables.

Question 2

Derive the truth value of $A$, given the following logical facts in a Knowledge Base:

- $B \Rightarrow \neg C$
- $A \lor B$
- $C \lor D$
- $\neg D \land E$. 

[10]
Question 3

You are given the following predicate vocabulary, where $x$ and $y$ are parameters:

- `AuthorOf(x, y)`
- `IsAuthor(x)`
- `IsProgram(x)`
- `Equals(x, y)`

Write down first-order logic sentences to express the following:

1. All programs have an author.
2. Richard Stallman wrote the program `emacs`.
3. At least one program has exactly three authors.
4. No programs were written by George.

Question 4

You are given the following joint probability table for binary random variables $A$, $B$, and $C$.

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>True</td>
<td>True</td>
<td>True</td>
<td>0.288</td>
</tr>
<tr>
<td></td>
<td>True</td>
<td>True</td>
<td>False</td>
<td>0.192</td>
</tr>
<tr>
<td></td>
<td>True</td>
<td>False</td>
<td>True</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td>True</td>
<td>False</td>
<td>False</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>False</td>
<td>True</td>
<td>True</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td>False</td>
<td>True</td>
<td>False</td>
<td>0.140</td>
</tr>
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<td>0.060</td>
</tr>
<tr>
<td></td>
<td>False</td>
<td>False</td>
<td>False</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Answer the following questions (showing all working):

1. Are $C$ and $A$ independent?
2. Is $B$ independent of $A$ given $C$?

Question 5

You are given the following Bayes Net.
Write down the CPTs you would need to fully describe this Bayes Net, and their sizes. (You do not need to fill in the CPTs, just write down what expression they represent, and the number of entries they have.)

Question 6

You are given the following Bayes Net:

\[ A \rightarrow B \rightarrow C \]

with \( P(A = \text{True}) = 0.6 \), and the following CPTs:

\[
\begin{array}{c|cc}
 B & A & P \\
 True & True & 0.6 \\
 False & True & 0.4 \\
 True & False & 0.1 \\
 False & False & 0.9 \\
\end{array}
\]

\[
\begin{array}{c|cc}
 C & B & P \\
 True & True & 0.25 \\
 False & True & 0.75 \\
 True & False & 0.5 \\
 False & False & 0.5 \\
\end{array}
\]

(a) (b)

Draw 3 samples from the joint distribution. Show your working.