CS141 **Introduction to AI** **Professor Meinolf Sellmann**

Homework 7

More Predicate Logic (and some DT stuff)

Due: 6:00PM on 11/11/09

Problem 7.1

A finite probability space, $(\Omega, P, 2^{\Omega})$ [1], where $P : 2^{\Omega} \rightarrow [0, 1]$ has the following properites:

a. $P(\emptyset) = 0$

b. $P(\Omega) = 1$

c. $P(A \cup B) = P(A) + P(B \backslash A)$

Conditional probability is defined as the probability of A given B and written as $P(A|B) := P(A \cap B)/P(B)$.

a. Let $P_B(A) = P(A|B)$. Show that $(B, P_B, 2^B)$ is a probability space.

b. Show that $P(A|B) = P(B|A) \cdot P(A)/P(B)$.

Solution:

Part A. P_B must satisfy the properties of a probabilitiy space enumerated above.

$$\begin{aligned} P_B(\emptyset) &= P(\emptyset|B) \\ &= \frac{P(\emptyset \cap B)}{P(B)} \\ &= \frac{P(\emptyset)}{P(B)} \\ &= 0. \end{aligned}$$

[1] 2^S where S is a set denotes the power set of S, which is the set of all subsets of S.

$$\begin{aligned} P_B(\Omega) &= P_B(B) \\ &= P(B|B) \\ &= \frac{P(B \cap B)}{P(B)} \\ &= \frac{P(B)}{P(B)} \\ &= 1. \end{aligned}$$

$$\begin{aligned} P_B(A \cup C) &= \frac{P((A \cup C) \cap B)}{P(B)} \\ &= \frac{P((A \cap B) \cup (C \cap B))}{P(B)} \\ &= \frac{P(A \cap B) + P((C \cap B) \setminus (A \cap B))}{P(B)} \\ &= P_B(A) + \frac{P((C \cap B) \setminus (A \cap B))}{P(B)} \\ &= P_B(A) + \frac{P((C \cap B) \setminus A) \cup ((C \cap B) \setminus B)}{P(B)} \\ &= P_B(A) + \frac{P((C \cap B) \setminus A)}{P(B)} \\ &= P_B(A) + \frac{P((C \setminus A) \cap B)}{P(B)} \\ &= P_B(A) + P_B(C \setminus A). \end{aligned}$$

Part B. By definition,

$$P(A|B) = \frac{P(AB)}{P(B)}$$

and

$$P(B|A) = \frac{P(BA)}{P(A)} = \frac{P(AB)}{P(A)}.$$

We therefore have that

$$P(AB) = P(B|A) \cdot P(A)$$

and so

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}.$$

Problem 7.2

Given the following axioms representing the group laws:

a. $\exists n, \forall a(f(n,a) = a = f(a,n))$

b. $\forall a, b, c(f(f(a,b),c) = f(a, f(b,c)))$

c. $\forall a \exists b(f(a,b) = n = f(b,a))$

Show that the inverse is unique.

Solution: Add the following to our knowledge base:

$$f(A, B_1) = n = f(B_1, A))$$

$$f(A, B_2) = n = f(B_2, A))$$
$$B_1 = B_1$$

For literals A, B_1, B_2.

Then via unification,

$B_1 = B_1$, $a = f(a,n)$ / $[a = B_1]$ $B_1 = f(B_1, n)$
$B_1 = f(B_1, n)$, $f(A, B_2) = n$ / $[n = f(A, B_2)]$ $B_1 = f(B_1, f(A, B_2))$
$B_1 = f(B_1, f(A, B_2))$, $f(a, f(b,c)) = f(f(a,b),c)$ / $[a = B_1, b = A, c = B_2]$ $B_1 = f(f(B_1, A), B_2))$
$B_1 = f(f(B_1, A), B_2))$, $n = f(B_1, A)$ / $[n = f(B_1, A)]$ $B_1 = f(n, B_2))$
$B_1 = f(n, B_2))$, $f(n,a) = a$ / $[a = B_2]$ $B_1 = B_2$

Problem 7.3

Unify the following expressions or state why they cannot be unified.

a. $in(X, Y)$ and $in(Z, office - of(Z))$

b. $in(X, X)$ and $in(Z, office - of(Z))$

c. $p(X, b, b)$ and $p(a, Y, Z)$

d. $p(Y, Y, b)$ and $p(Z, X, Z)$

e. $p(f(X, X), a)$ and $p(f(Y, f(Y, a)), a)$

Solution:

a. $X = z$, $Y = office - of(Z)$

b. Occurs check.

c. $X = a$, $Y = b$, $Z = b$

d. $X = Y$, $Y = Z$, $Z = b$

e. Occurs check.