HW2

Due: September 22, 2016

Attach a fully filled-in cover sheet to the front of your printed homework. Your name should not appear anywhere except on the cover sheet; each individual page of the homework should include your Banner ID instead.

While collaboration is encouraged in this class, please remember not to take away notes from any collaboration sessions. Also please list the names and logins of any of your collaborators at the beginning of each homework.

Please monitor Piazza, as we will post clarifications of questions there. You should hand in your solutions by 12:55 to the CSCI 1010 bin on the second floor of the CIT. Late homeworks are not accepted.

In general, if you submit a complicated, messy DFA/NFA and include no explanation of how it works, it will probably not be graded.

Problem 1

The O’Nion family has purchased some new root-vegetable-analyzing machines with the profits from their mangelwurzel empire. Their daughter Ginger has just graduated with a degree in engineering from Rootgers University. They ask her to look at the machines (which she knows to be NFAs) and to determine which type of crop is recognized by each machine. The alphabet is \{0, 1\}.

What languages do the following NFAs recognize?

Note: A state with two circles denotes an accept state.
Problem 2

To stay competitive with the rival O’Nion family, the Begas seek similar machines as those in Problem 1 to help them better understand their potato yields. Lacking the money to purchase the machines, however, they enlist their Farmherst-educated, tech-savvy son, Rudolph, to construct the machines for them.

Help Rudolph set up DFAs for the following languages over the alphabet $\Sigma = \{0, 1\}$.

a. $\{w \mid w$ is a binary representation of a power of two$\}$

b. $\{w \mid w$ starts with 0 and has an even number of 0s$\}$

c. $\{w \mid w$ starts with 0 and every 1 is followed by a 0$\}$
Problem 3

Convert the following NFA to a DFA. Note that it may be useful to look for shortcuts along the way rather than simply brute-forcing a solution.

The following questions are lab problems.

Lab Problem 1

Construct an NFA over the alphabet $\Sigma = \{0, 1\}$ that recognizes the language of binary strings $w$ such that $w$ is either three times a power of two or is divisible by five when interpreted as a binary integer with its most significant bits first.

Note: Your NFA should be as simple as possible. Also please provide a brief explanation of why your NFA works.
Lab Problem 2

a. Suppose that $L_1$ and $L_2$ are regular languages. Define their intersection, $L_1 \cap L_2$, as $\{w \mid w \in L_1, w \in L_2\}$. Prove that $L_1 \cap L_2$ is regular.

b. Suppose that $L$ is a regular language. Define $L_r$ as $\{w = w_1w_2w_3...w_n \mid w_nw_{n-1}...w_1 \in L\}$. Prove that $L_r$ is regular. **Hint:** Use (D/N)FAs as the root of your proof.

Lab Problem 3

In an effort to outdo one another, the O’Nions and the Begas begin to experiment with genetically engineered crops. Having both become masters of carrots and potatoes, they seek to splice the two roots’ genes together to form the rootiest of vegetables. But while both families completely understand the DNA of the individual crops, they have different approaches for how to combine them.

Ginger believes that for maximal mixing, the individual nucleotides of the two vegetables should alternate. Thus Ginger is only interested in nucleotide sequences of the form $c_1p_1c_2p_2...c_np_n$, where $c_1c_2...c_n$ and $p_1p_2...p_n$ are full carrot and potato genes, respectively.

Rudolph, however, is much less picky. He still wants to splice the two genes, but they don’t have to alternate individual nucleotides. Instead, he is interested in all sequences of the form $C_1P_1C_2P_2...CnP_n$, where $C_1, C_2,..., C_n$ are each strings of nucleotides for carrot genes, and $P_1, P_2,..., P_n$ are each strings of nucleotides for potato genes. He still requires that $C_1C_2...C_n$ is a full carrot gene, and $P_1P_2...P_n$ is a full potato gene.

Both Ginger and Rudolph already have DFAs that recognize full carrot and potato genes. They want to build DFAs that will recognize whether a potential hybrid gene sequence is valid according to their respective definitions.

a. Prove that Ginger’s machine can be built. In other words, given a DFA for some languages *Carrot* and *Potato*, prove that you can construct a DFA recognizing the language:

$$L = \{w \mid w = c_1p_1...c_np_n, \text{ where } c_1...c_n \in \text{Carrot} \text{ and } p_1...p_n \in \text{Potato},$$
and each $c_i, p_i$ are single characters in $\Sigma\}$$

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b. Prove that Rudolph’s machine can be built. In other words, given a DFA for Carrot and Potato, prove that you can construct a DFA for:

\[ L = \{ w \mid w = C_1P_1 \ldots C_nP_n, \text{ where } C_1 \ldots C_n \in \text{Carrot and } P_1 \ldots P_n \in \text{Potato,} \]

and each \( C_i, P_i \) are strings over \( \Sigma \), potentially empty\]