HW10

Due: November 30, 2017

Attach a fully filled-in cover sheet to the front of your printed homework. Your name should not appear anywhere; the cover sheet and each individual page of the homework should include your Banner ID only.

While collaboration is encouraged in this class, please remember not to take away notes from any labs or collaboration sessions.

Please monitor Piazza, as we will post clarifications of questions there. Hand in your solutions by 12:55 to the CSCI 1010 bin on the second floor of the CIT. Late homeworks are not accepted.

Problem 1

Use Rice’s theorem to show that the following languages are undecidable:

a. \( L_{\text{inde}} = \{ \langle M \rangle \mid M \text{ is a TM and } |L(M)| \geq 1 \} \)

b. \( L_{\text{arch}} = \{ \langle M \rangle \mid M \text{ is a TM and } “larch” \in L(M) \} \)

Problem 2

Last week at the Game of Cones, Will Owens found some suspicious subsequences in the Discrete Logging Company’s conifer promotional materials. For some reason, all of the materials are signed by someone named Holly Pine—but Will has never heard of Holly. One of the documents describes a Turing machine and a language Will never thought was possible to decide.

In order to get to the bottom of these shady goings-on, help Will with the following language:

\[ L = \{ \langle M, k \rangle \mid M \text{ is a TM that halts on at least } k \text{ inputs} \} \]

Is this language recursively enumerable? If so, is it decidable? Prove your answers.

**Hint:** You may want to use nondeterminism.
Problem 3

Prove that the following languages are decidable:

a. \( L_a = \{ \langle Q \rangle \mid Q \text{ is a DFSM and the language of } Q \text{ is infinite} \} \)

b. \( L_b = \{ \langle Q, R \rangle \mid Q \text{ and } R \text{ are DFSMs with } L(Q) \subseteq L(R) \} \)

c. \( L_c = \{ \langle Q, S \rangle \mid Q \text{ is a DFSM, } S \text{ is a finite set of strings, and } L(Q) = S \} \)

The following questions are lab problems.

Lab Problem 1

In her spare time, U. K. Liptus likes to imagine running Turing machines within Turing machines. She has made a lot of progress on the laurel-shea tree hybrids, and they are the most environmentally conscious trees the Discrete Logging Company has ever produced. The obvious next step is to make them conscious.

Making the laurel-shea trees conscious ends up being harder than she expected. What is consciousness anyway? Is it possible to recognize consciousness? Taking a break from these questions, U. K. Liptus goes back to her Turing machine problems.

She’s interested in Turing machines that are self-terminating, meaning a machine that halts when given its own description as input.

a. Prove that the language \( L_{\text{NST}} = \{ \langle M \rangle \mid M \text{ is not self-terminating} \} \) is undecidable. You may not use Rice’s theorem.

b. Prove that the language \( L_{\text{NST}} \) is not recursively enumerable.
Lab Problem 2

Meanwhile, at a secret conclave of the Illuminatree, the mysterious Holly Pine is working on innovations that will help destroy the logging competition in Yew Nork. A little-known fact about the Pinary Tree Company is that Douglas and Connie Ferr only took control after the Pine family, the founders and namesakes of the company, disappeared in an accident years ago. The story has become something of an urban legend, so little do they suspect that Holly Pine or anyone associated with the Pines might come back.

Holly Pine has since changed her name and slowly worked her way up Yew Nork’s arboreal economy. Along the way, she has amassed a group of friends who support her claim to the Pinary Tree Company. Known to outsiders as the Illuminatree, they hope to bring more biodiversity to Yew Nork in the form of non-conifer trees and other tree hybrids. In order to make room for their trees, they need to correctly model growth and logging activities, in the form of new Turing machines.

a. A tree trimmer is a TM that ‘trims’ all words of length $k$ or greater from the language of an input TM. That is, on input $\langle M_1, k \rangle$, a tree trimmer outputs a TM $\langle M_2 \rangle$ such that $w \in L(M_2)$ if and only if $w \in L(M_1)$ and $|w| < k$, where $k \geq 0$.

Let $\text{TreeTrimmers} = \{ \langle T \rangle \mid T \text{ is a tree trimmer} \}$. Is $\text{TreeTrimmers}$ recursively enumerable? Prove your answer.

b. In her quest to grow ever taller trees, Holly Pine has modified a tree trimmer, creating a tree skimmer. This new variety ‘skims’ all words of length $k$ or less from the language of an input TM. That is, on input $\langle M_1, k \rangle$, it outputs a TM $\langle M_2 \rangle$ such that $w \in L(M_2)$ if and only if $w \in L(M_1)$ and $|w| > k$, where $k \geq 0$.

Let $\text{TreeSkimmers} = \{ \langle T \rangle \mid T \text{ is a tree skimmer} \}$. Is $\text{TreeSkimmers}$ recursively enumerable? Prove your answer.
Lab Problem 3

Recall that in the decision problem 3SAT, we wanted to determine whether there was an assignment of truth values that satisfied every clause of a 3CNF Boolean formula. In the optimization version Max3SAT, the goal is to find some assignment of truth values that satisfies the maximum possible number of clauses, even if it is not possible to satisfy every single clause.

Design a polynomial-time algorithm that, given as input a Boolean formula in 3CNF, finds an assignment of values that satisfies at least 7/8’s as many clauses as the maximum possible number of satisfiable clauses. You should assume that each clause contains exactly 3 literals of distinct variables.

**Hint:** The algorithm should set each variable in the formula one by one, depending on the expected number of satisfied clauses given a particular assignment. That is, the algorithm should choose an assignment to variable $x_i$ based on the expected number of satisfied clauses given $x_i = 1$ and the expected number of satisfied clauses given $x_i = 0$. 