Attach a fully filled-in cover sheet to the front of your printed homework. Your name should not appear anywhere; the cover sheet and each individual page of the homework should include your Banner ID only.

While collaboration is encouraged in this class, please remember not to take away notes from any labs or collaboration sessions.

Please monitor Piazza, as we will post clarifications of questions there. Hand in your solutions by 12:55 to the CSCI 1010 bin on the second floor of the CIT. Late homeworks are not accepted.

Problem 1

Give symbolic descriptions of the following languages by using concatenation, Kleene star, $n$-fold Cartesian products, and union. Let the alphabet be $\Sigma = \{a, b, c, d\}$. You may explicitly specify any finite sets.

a. Strings where no ‘c’ appears earlier than any ‘a’.

b. Strings where every ‘d’ is immediately followed by either ‘a’ or ‘cbc’.

c. Strings in which either no letter or every letter is ‘b’.

d. Strings of length less than 5 or of length greater than 9.
Problem 2

**Conjunctive Normal Form (CNF)** is a particular way of writing Boolean formulae that will be used in this course. We define it constructively:

1. A **literal** is a variable or its complement. Two examples of literals are $x_i$ and $\neg x_i$.
2. A **clause** is any disjunction (OR) of any number of literals. A literal by itself is also a clause.
3. A formula is in **Conjunctive Normal Form** if it is the conjunction (AND) of any number of clauses.

As an example, the formula $(x_1 \vee x_3) \land (\neg x_1 \land x_2) \land \neg x_2$ is in CNF. Using the above definition, convert the following formulae into Conjunctive Normal Form:

a. $\neg(x_1 \land x_2) \land \neg(\neg x_1 \land x_3)$

b. $(x_1 \land x_4) \lor (x_2 \land x_3) \lor (x_1 \land x_3)$

c. $(x_1 \lor x_2) \land (x_1 \oplus x_3)$

d. $\neg(x_1 \to (x_2 \to (x_3 \to x_4)))$

e. $x_1 \oplus x_2 \oplus x_3$

Problem 3

The city of Yew Nork is the world’s largest producer of conifer trees. Arborists classify conifers into seven different families including pine, yew, and cypress. Help them verify that the equivalence relation of “being in the same conifer family” describes nonoverlapping families.

That is, let $R \subseteq A \times A$ be an equivalence relation and let $E[a]$ denote the set of elements in $A$ equivalent to the element $a$ under the relation $R$. Show that for all $a, b \in A$, the equivalence classes $E[a]$ and $E[b]$ are either equal or disjoint.
CSCI 1010 Due: September 14, 2017

The following questions are lab problems.

Lab Problem 1

This problem considers a graph $G = (V, E)$. First, some terminology:

- A **clique** is a subset of vertices $S \subseteq V$ such that $\forall u, v \in S, (u, v) \in E$. That is, every pair of vertices in $S$ is connected by an edge in $E$.

- A **vertex cover** is a subset of vertices $S \subseteq V$ such that for any edge $(u, v) \in E$, $u \in S$ or $v \in S$.

- An **independent set** is a subset of vertices $S \subseteq V$ such that $\forall u, v \in S, (u, v) \notin E$.

- The **graph complement** is a graph $\overline{G} = (V, \overline{E})$ on the same vertices such that $(u, v) \in \overline{E}$ if and only if $(u, v) \notin E$.

Consider a graph $G = (V, E)$.

a. Show that a subset of vertices is an independent set in $G$ if and only if it is a clique in $\overline{G}$.

b. Show that a subset of vertices $S \subseteq V$ is an independent set if and only if $V \setminus S$ is a vertex cover.

Lab Problem 2

One of the hottest startups in Yew Nork, the Discrete Logging Company, is designing a new circular arboretum. They need to divide the arboretum into regions for each species of conifer they will plant.

Use induction to show that with $n$ straight-line cuts, they can divide the circle into $\frac{n^2 + n + 2}{2}$ regions, but no more. Some trees require more space than others, so the regions may be of unequal shape and size.
Lab Problem 3

The Pinary Tree Company is a competing forestry organization in Yew Nork. Its leaders, Douglas and Connie Ferr, are investing in research for their circuitree department. They’ve hired you to prove some preliminary results about circuit satisfiability.

Consider a circuit with $n$ input wires and one output wire. Let the set of the circuit’s input wires be $I_n = \{w_1, w_2, \ldots, w_n\}$ where $w_i$ represents the $i^{th}$ wire. Consider some subset of input wires $t \subseteq I_n$. We say that the subset satisfies the circuit if the circuit evaluates to True when all input wires in $t$ are set to True and all input wires not in $t$ are set to False.

Let $T_n$ be the set of all $t \subseteq I_n$ such that $t$ satisfies the circuit. Prove that for any $n \geq 2$, there exists a circuit where:

a. $|T_n| = 1$

b. $|T_n| = 2^n - 1$

c. $|T_n| = 2^n$

d. $|T_n| = 2^{n-1}$

e. $|T_n| = n$

You do not need to use induction or draw circuit diagrams for this problem.