Post correspondence problem (PCP):

$$PCP = \{ \langle [t_1], [t_2], \ldots, [t_n] \rangle \mid \exists a \text{ sequence } t_1, \ldots, t_k \text{ s.t. } t_1 \odot t_2 \odot \ldots \odot t_k = b_1 \odot b_2 \odot \ldots \odot b_k \}$$

Ex. $$[b], [a^2], [ca], [abc] \Rightarrow$$

Solution: $$[a \mid b \mid c \mid a \mid a \mid abc]$$

top and bottom are both abcacaabac

Ex2 $$[aaa^n], [aa^n], [nn]$$

no solution

So, $$EX \cap PCP$$, $$EX \notin PCP$$

$$MPCP$$: same as PCP, but $$i_i = 1$$. Equivalent problem because if the set of solutions to PCP can be reordered so the first domino in the solution is also $$[b_1]$$, other direction proved on next page.

Thm: $$ATM = MP$$

BE: Must give a computable function $$f$$ s.t. $$\langle M, W \rangle \in ATM \iff f(\langle M, W \rangle) \in MP$$

Roadmap: dominoes line up with computation history of $$M$$ on $$W$$

1. First domino corresponds to the start configuration of $$M$$ on $$W$$. In the ex:

$$\begin{bmatrix}
\# & \# & \# & 0 & 0 & \ldots & \# & \# & \# & \# \\
0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
\end{bmatrix}$$

2. Corresponding to a transition $$\delta(q, a) = (r, b, R)$$:

Create $$[q \ a \ b \ r \ \#]$$

in ex:

$$\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & \ldots & 1 & 0 & 1 & 0 \\
\end{bmatrix}$$

(DFA changes into TM, always moves right)

3. Similar for left -> see next page
\[ \forall a \in \Gamma, \text{ make } [\hat{a}] \]
\[ \text{ in } \begin{bmatrix} 0' & 1' & \vdash \\end{bmatrix} \]
\[ \begin{bmatrix} \# & \# \end{bmatrix} \]
\[ \forall a \in \Gamma, \text{ make } [\hat{a} \#] \]
\[ [\hat{a} \#] \]
\[ \begin{bmatrix} \# & \# \end{bmatrix} \]

For more left: \[ \text{ if } \{(q,a) = (r,b)\}, \text{ for any } c \in \Gamma, \text{ add } [\hat{c} \hat{q} \hat{a}] \]
\[ \{r \leq c \leq b\} \]

Thm: \[ mPCP \leq mPCP \]
\[ \begin{bmatrix} t_1 \vdash \end{bmatrix} \Rightarrow \begin{bmatrix} \hat{a}_1 \hat{a}_2 \ldots \hat{a}_k \vdash \end{bmatrix} \]
\[ \begin{bmatrix} \hat{b}_1 \hat{b}_2 \ldots \hat{b}_l \vdash \end{bmatrix} \]
\[ \begin{bmatrix} \hat{t}_i \vdash \hat{a}_1 \hat{a}_2 \ldots \hat{a}_k \hat{b}_1 \hat{b}_2 \ldots \hat{b}_l \vdash \end{bmatrix} \]
\[ \text{ add } \begin{bmatrix} \# \hat{a}_1 \hat{a}_2 \ldots \hat{a}_k \hat{b}_1 \hat{b}_2 \ldots \hat{b}_l \# \end{bmatrix} \]

Computable function \( f \): On input dominoes \( \{x_i : y_i\}_{i=1}^n \), modify by adding *'

- First domino: top & bottom start \( u/ \# \), insert * between any pair of symbols, add one to bottom end of bottom.
- \( i \geq 1 \), \( \text{ith} \): top starts \( v/ \# \)
  - insert * like \( i=1 \)
  - add \( \begin{bmatrix} \# \hat{v} \end{bmatrix} \)
  - bottom ends \( w/ \# \)
AllCFG

\[ \text{AllCFG} = \{ \langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^* \} \]

Thm: AllCFG is not Turing-reducible

PF: \( U \oplus \text{ATM} \leq_T \text{AllCFG} \)

Must give computable \( f \) s.t.

\( \hat{M} \) accepts \( w \) if \( f(\langle M, w \rangle) \) generates all strings over \( \Sigma \)

Idea: \( G \) generates all strings that are not an accepting CH of \( M \) on \( w \)

What does CH look like?

\[
\begin{align*}
\# & \cdots \# \\
\hat{C}_0 & \hat{C}_1 \cdots \hat{C}_r \\
\text{blocks} & \\
\Sigma = \{0, 1, \#, \} \\
\end{align*}
\]

\[
\begin{align*}
S_1: & \text{strings that don't start or end with } \# \\
S_2: & \# \cdots \# \text{ (no intermediate } \# \text{'s)} \\
S_3: & \# \hat{C}_0 \# \cdots \# \text{ but } \hat{C}_0 \neq \# \hat{C}_0 \# \cdots \# \\
S_4: & \# \cdots \# \hat{C}_r \# \cdots \# \text{ but } \hat{C}_r \text{ does not contain } \# \hat{C}_i \# \cdots \# \text{ from } \hat{C}_i \\
S_5: & \text{not thoroughly shown in class.}
\end{align*}
\]