You can encode a DFA, CFG, TM in binary.

- If you get a binary string that isn't a proper def of DFA, default to interpreting as DFA that rejects everything.

So consider the language $A_{DFA} = \{ <M, w> \mid M \text{ is a DFA that accepts } w \}$

**Is $A_{DFA}$ decidable?**

**YES** — Run $M$ on $w$, accept if it acc., rej. o.w.

**Is $A_{CFG}$ decidable?**

**YES** — Convert $G$ to Chomsky NF.

Try every derivation of len. $2n-1$ ($n=|w|$)

If one ends in $w$ accept, else rej.

**Is $A_{TM}$ decidable?**

**NO** — But it is *Turing recognizable* using same template as for $A_{DFA}$.

**Proof:** Suppose $A_{TM}$ were decidable, and let $D$ be a decider for $A_{TM}$.

Then consider the TM $C$:

On input $<M>$,

Run $D$ on input $<M, <M>>$
If $D$ outputs $\text{acc}$, you reject.
Else, accept.

$C$ can’t be anywhere on the table ($\text{table} = \text{TM} \times \text{TM}$).
But $\text{table}$ is all TMs, so $\text{TM}$.

**Alternative approach:**

What does $C$ do on input $\langle c \rangle$?

- Suppose it accepts.
  Then $D(\langle c, \langle c \rangle \rangle)$ accepts.
  But then $C$ rejects, so $C$ cannot accept $\langle c \rangle$.
- Suppose it does not accept $\langle c \rangle$.
  Then $D(\langle c, \langle c \rangle \rangle)$ rejects.
  Then $C$ accepts. So $C$ can’t accept $\langle c \rangle$. $\blacksquare$

**What about $\text{TM}^c$?**

**NO** - For any language $L$, if $L$ is not decidable then we know $L^c$ is not decidable.

(B/c if $L^c$ were decidable we could define $L$ as what rej. by $L^c$)

But is it Turing-recognizable?
NO - We know $Atm$ is Turing-rec. So if $Atm^c$ were Turing-rec, we could put them together and make a decider.

**Inm**: $L$ is decidable iff both $L$ and $L^c$ are Turing-recognizable.

Suppose $L$ is decidable, let $M$ be a TM deciding $L$, then $L = L(M)$, so it's Turing-rec.
Consider $M'$, identical to $M$ but with $q_a$ and $q_b$ switched. $L^c = L(M)$, so it's Turing-rec as well.

Suppose $L$ and $L^c$ are Turing-rec. Let $M$, $M'$ be TM's st. $L = L(M)$ and $L^c = L(M')$.
Consider D:
On input $w$,
\[
\begin{array}{l}
\text{Run } M \text{ on input } w \quad \text{In parallel} \\
\text{Run } M' \text{ on input } w
\end{array}
\]
If $M$ accepts, accept.
If $M'$ accepts, reject.