Attach a fully filled-in cover sheet to the front of your printed homework. Your name should not appear anywhere; the cover sheet and each individual page of the homework should include your Banner ID only.

While collaboration is encouraged in this class, please remember not to take away notes from any labs or collaboration sessions.

Please monitor Piazza, as we will post clarifications of questions there. Hand in your solutions by 10:20am to the CSCI 1010 bin on the second floor of the CIT. Late homeworks are not accepted.

Problem 1

Has anyone really been far even as decided to use even go want to do look more like?

After contemplating this life-altering question, you decide to ponder some more open-ended questions.

Are P, NP, and PSPACE closed under the complementation operation? Also, is PSPACE closed under the Kleene star operation? For each of your answers, either prove it or relate it to an open question.

Problem 2

a. Let $A$ be the language of matching parentheses. For example, “(())" and “((()))()" are in $A$, but “)" is not. Show that $A$ is in L.

b. Let $B$ be the language of matching parentheses and brackets. For example, “[()]" and “[(())()][]" are in $B$, but “[)]" is not. Show that $B$ is also in L.

The following questions are lab problems.
Lab Problem 1

A ladder is a sequence of strings $s_1, \ldots, s_k$ such that every string differs from the preceding string by exactly one character and all strings are of the same length. For example, the following is a ladder of English words, starting with “head” and ending with “free”: head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free. More formally,

$$LADDER = \{ \langle D, s, t \rangle \mid D \text{ is a DFA and } L(D) \text{ contains a ladder of}$$

strings, starting with $s$ and ending with $t\}$$

1. Give a ladder from ‘dank’ to ‘meme’ using only common English words. (No proper nouns allowed!)

2. Show that $LADDER \in PSPACE$.

Word ladders are super fun! Here are some more for you to try: George to Claire, May to Ken, Shiv to Anna, Mac to Sam, Mina to Matt. Remember, no proper nouns are allowed. The CS1010 staff makes no claims as to whether any of these are possible; we leave that as an exercise to the reader.

Lab Problem 2

Remember Ascii from HW9? The carpenter tasked with the immensely important task of constructing tiny digital tables so that internet users from all around the world can flip tables to their heart’s content?

$$\text{Figure 1: Ascii in action.}$$

His tables are now being used as a part of an incredibly popular two-player online game called TABLETFIGFLIP. The rules of the game are as follows:

- Recall that each tabletop has some holes that align with a two-column, $N$-row grid.
- Each player starts with their own ordered stack of tabletops.
- Players then take turns stacking tabletops in a tabletop-sized crate, and are allowed to flip (i.e. rotate about the vertical axis) the tabletops before placing them in the crate.
• The game ends when both players have used up all of their table tops. Player 1 wins if all the hole positions are blocked in the final stack, and Player 2 wins if some hole position remains unblocked.

• Player 1 has the first turn.

Show that the problem of determining whether Player 1 has a winning strategy for a given starting configuration of the table tops is PSPACE-complete.

**Hint:** Recall that in our previous encounter with Ascii’s tabletops in HW9, we reduced from 3SAT to show NP-hardness. To do so, we had a tabletop for every variable $x$ and a row for every clause. For each tabletop, we put a hole in the left column of row $i$ if the variable appeared as $x$ in the $i$th clause, we put a hole in the right column if the variable appeared as $\neg x$ in the $i$th clause, and we put a hole in both columns if the variable did not appear in the $i$th clause. We also included a tabletop with holes all the way down the left column. With this poly-time reduction, we showed that a 3CNF formula was satisfiable if and only if it was possible to arrange the linings so that every hole was covered.
Lab Problem 3

Having brought eternal shame upon his small nation at the Olympic Games, Qwop decides to quit track, and instead decides to focus his energies on a game called LOSINGSUBSET customary to his small nation.

The game is based on a directed graph, and the rules are as follows:

- At the beginning of the game, a certain node $s$ of the graph $G$ is colored green.
- The two players take turns picking a neighboring uncolored node to the green node. It becomes the new green node, and the previous green node is now colored red. (If the green is at node $u$, it can only move to node $v$ if $(u,v) \in E$, $v$ is not red, and $v \neq u$).
- There is a subset of nodes $L \subset V(G)$. The first player to move onto a vertex that belongs to the subset $L$ loses, and the other player wins. ($L$ does not contain the starting node $s$.)
- If the current player cannot make a move (because all neighboring nodes have been visited), and no one has lost yet, the game ends in a tie.

Show that determining whether the first player has a winning strategy in an arbitrary instance of the game LOSINGSUBSET is PSPACE-complete. A tie is not a win. Note that an instance of the game LOSINGSUBSET consists of a directed graph $G$, a starting vertex $s$, and a losing subset $L$.

**Hint:** Recall from class the PSPACE-complete problem GENERALIZED-GEOGRAPHY. To remind you, GENERALIZED-GEOGRAPHY is played in the same manner as LOSINGSUBSET, with two players taking turns moving a pebble on a directed graph. A player loses if they cannot move the pebble anywhere (i.e. if all neighboring nodes have already been visited).