Midterm 1

Due: October 28, 2021, 11:59 PM ET

Reminder: Your name should not appear anywhere on your handin; each individual page of the homework should include your Banner ID only. For your digital submission, each page should include work for only one problem (i.e., make a new page/new pages for each problem).

Collaboration or any kind or use of any third-party resource is strictly not allowed on this assignment. The TA staff will only answer questions regarding clarifications on the statement of the problems in the assignment. Questions should be asked via private posts on Ed. Please monitor Ed, as we will post clarifications of frequently asked questions there.

Hand in your solutions by 11:59pm to Gradescope. See the course syllabus for the late policy. Usage of any materials outside of course notes, the course textbook, lecture slides and Ed posts is strictly forbidden.

Problem 1 is worth 10 points, Problem 2 is worth 10 points, Problems 3 is worth 20 points and Problem 4 is worth 20 points. Best of luck!

You may apply up to two late days to this assignment.

Problem 1

Convert the following DFAs to regular expressions.

a.

b.
Problem 2

For any strings $w$ and $t$, let $\text{substring}(w) = \{t \text{ IFF } t \text{ is a substring of } w\}$. Let $A$ be a regular language. Let the transformation $T$ on regular language $A$ result in the new language $L$ such that $L = \{b | a \in A \text{ and } \text{substring}(a) = b\}$. Prove that regular languages are closed under $T$.

Problem 3

Here we introduce the notion of a queue automaton. The concept is similar to a PDA, but instead of utilizing a stack, it utilizes a queue. That is, we can perform “push” and “pop” primitives with the transition between any two states. The push operation puts the ingested terminal at the top of the queue, and the pop operations removes whichever terminal is at the bottom of the queue.

Recall that to prove that a computational model $A$ is more powerful than another model $B$, it must be proven that (1) $A$ is not less powerful than $B$ and (2) there is some language that can be recognized by $A$ that $B$ cannot recognize.

Use the following problems to prove that a queue-automaton is more powerful than a PDA.

$Hint$: Utilize non-determinism to help with your solution!

a. Show that a queue-automaton is not less powerful than a PDA.

b. Find a language that cannot be recognized by a PDA, but that can be recognized by a queue-automaton. Then, from your answers to this part and the previous, conclude that queue-automata are more powerful than PDAs. $Hint$: think of one of the languages that we discussed in class not being CFL.
Problem 4

Give a description, in English, of a Turing Machine that recognizes the following Turing-decidable language.

\[ L = \{ a^x b^y c^z \mid y \% x = z \}, \]

where \( \% \) is the modulo operator.

(Note: the description should be detailed enough that your graders can unambiguously understand what your TM is doing :-) )