Reminder: Lab problems are graded and must be submitted along with homework when lab writeups are due. Your name should not appear anywhere on your handin; each individual page of the homework should include your Banner ID only. For your digital submission, each page should include work for only one problem (i.e., make a new page/new pages for each problem).

While collaboration is encouraged in this class, please remember not to take away notes from any labs or collaboration sessions.

Please monitor Piazza, as we will post clarifications of questions there. Hand in your solutions by 10:20am to Gradescope. Late homeworks are not accepted.

The following questions are lab problems.

Lab Problem 1

A ladder is a sequence of strings $s_1, \ldots, s_k$ such that every string differs from the preceding string by exactly one character and all strings are of the same length. For example the following is a ladder of English words, starting with “head” and ending with “free”: head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free. More formally,

$LADDER = \{(D, s, t) \mid D \text{ is a DFA and } L(D) \text{ contains a ladder of strings, starting with } s \text{ and ending with } t\}$

1. Give a ladder from ‘dank’ to ‘meme’ using only common English words. (No proper nouns allowed!)

2. Show that $LADDER \in PSPACE$.

Word ladders are super fun! In the spirit of our heavily-unupdated course assignment theming, here are some more for you to try, inspired by the CS1010 TA staff in Fall 2018: George to Claire, May to Ken, Shiv to Anna,
Lab Problem 2

Legendary Chef Boyardee has “accidentally” poisoned a leading government official who was pushing a ravioli ban. While he has been spared execution by stewing, he unfortunately has to spend the rest of his life in prison. In order to fill his time, he has developed an interest in a new game popular among his cellmates. The game involves constructing an acyclic graph called a superconcentrator with the properties described below.

An $n$-superconcentrator is a directed acyclic graph $G = (V, E)$ with $n$ input vertices and $n$ output vertices such that for any $r$ inputs and $r$ outputs, with $1 \leq r \leq n$, there are $r$ vertex-disjoint paths in $G$ connecting these inputs and outputs. Paths are vertex-disjoint if they have no vertices in common.

a. For $S + 1 \leq n$, prove that to pebble any $S + 1$ outputs on an $n$-superconcentrator from an initial placement of $S$ pebbles requires that at least $n - S$ different inputs be pebbled.

b. Use the result of part (a) to show that to pebble an $n$-superconcentrator with $S$ pebbles in time $T$ requires $S$ and $T$ to satisfy the following inequality:

$$(S + 1)T \geq \frac{n^2}{2}$$

Hint: Divide time up into consecutive intervals, chosen so that each interval has the same number of outputs pebbled during it. Apply the results of part (a) to obtain a lower bound on the sum of the number of input and output vertices that are pebbled during the interval.