HW7
Due: October 31th, 2019

Reminder: Lab problems are graded and must be submitted along with homework when lab writeups are due. Your name should not appear anywhere on your handin; each individual page of the homework should include your Banner ID only. For your digital submission, each page should include work for only one problem (i.e., make a new page/new pages for each problem).

While collaboration is encouraged in this class, please remember not to take away notes from any labs or collaboration sessions.

Please monitor Piazza, as we will post clarifications of questions there. Hand in your solutions by 10:20am to Gradescope. Late homeworks are not accepted.

Problem 1
For each of the following languages, explain whether it is Turing-recognizable.

a. \( L = \{ \langle M \rangle \mid M \text{ is a TM and } \text{trololol} \in L(M) \} \).

b. \( L = \{ \langle M, w, s \rangle \mid M \text{ at some point writes symbol } s \text{ on the tape given input } w \} \)

c. \( L = \{ \langle M \rangle \mid |L(M)| \leq 9000 \} \)

Problem 2
For each of the following languages, determine whether it is decidable and recognizable, undecidable but recognizable, or unrecognizable. Prove your answer.

a. \( L_a = \{ \langle M, k \rangle \mid M \text{ is a TM that halts on at least } k \text{ inputs} \} \)

b. \( L_b = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs and } |L(M_1)| = |L(M_2)| \} \)

c. \( L_c = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are DFAs with } L(M_1) \subseteq L(M_2) \} \)
Problem 3

Consider the following language:

\[ L = \{ \langle M \rangle \mid M \text{ accepts input } \varepsilon \} \]

We wish to prove that \( L \) is undecidable. Recall that we know that

\[ A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \} \]

is undecidable.

Candidate Proof: Assume there exists a decider \( A \) for \( A_{TM} \). Then we construct the following decider \( D \) for \( L \): \( D \) takes input \( \langle M \rangle \) and runs \( A \) on \( \langle M, \varepsilon \rangle \), then outputs whatever \( A \) returns. Clearly, \( D \) will halt because it is only running \( A \), which is a decider. \( D \) also clearly decides \( L \), because it only returns true when \( M \) accepts \( \epsilon \), and false otherwise. However, this is a contradiction, since \( A_{TM} \) is not decidable, so \( L \) must not be decidable.

1. What is wrong with the proof given above? Explain.
2. How would you correctly prove that \( L \) is undecidable?