Problem 1

Consider the following two languages. For each language, determine whether you can use Rice’s Theorem to prove it is undecidable. If so, use Rice’s Theorem to prove it is undecidable. If not, explain why you cannot use Rice’s Theorem, and prove it is undecidable without using Rice’s Theorem.

a. \( L_{yue} = \{ \langle M \rangle \mid |L(M)| \geq 1 \} \)

b. \( L_{cek} = \{ \langle M \rangle \mid \langle M \rangle \in L(M) \} \)

Problem 2

Explain why each of the following languages is decidable:

a. \( L = \{ \langle M, w \rangle \mid \text{when } M \text{ is run on } w, \text{ it only moves left} \} \)

b. \( L = \{ \langle M, w \rangle \mid \text{when } M \text{ is run on } w, \text{ the head reverses direction at least once} \} \)

Note: When a Turing machine moves left at the start of the tape, it just stays at the start of the tape.

Hint: Consider how you would detect an infinite loop, given that you haven’t changed direction.
Problem 3

Consider the following language:

\[ L = \{ \langle M, k \rangle \mid M \text{ is a TM that halts on at least } k \text{ inputs} \} \]

Is this language Turing-recognizable? If so, is it decidable? Prove your answers. (Hint: you may want to use nondeterminism.)