Problem 1 (paper hand-in): Let \( vlist \) consist of the following vectors with domain \( D = \{a, b\}: \)
\[
\text{Vec}(D, \{a:2, b:5\}), \text{Vec}(D, \{a:8, b:10\}), \text{Vec}(D, \{a:-4, b:12\}), \text{Vec}(D, \{a:1, b:-4\}).
\]
By running \text{orthogonalize} on this list, we get four vectors:
\[
\text{Vec}(D, \{a:2, b:5\}), \text{Vec}(D, \{a:3.45, b:-1.38\}), \text{Vec}(D, \{}), \text{Vec}(D, \{}).
\]
We write the relationship in terms of a matrix equation:
\[
\begin{bmatrix}
0 & 1 & 2 & 3 \\
2 & 8 & -4 & 1 \\
5 & 10 & 12 & -4 \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
\end{bmatrix}
= \begin{bmatrix}
0 & 1 & 2 & 3 \\
2 & 3.45 & 0 & 0 \\
5 & -1.38 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 2 & 3 \\
2.28 & 1.79 & -0.621 \\
1 & 0 & 1 & -2.2 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The columns of the first matrix are the original vectors, and the columns of the second matrix are the starred vectors.

1. Give a basis for the vector space spanned by the original vectors. Explain how you got the vectors forming the basis.

2. Use a procedure from the module \text{triangular} to find a basis for the null space of the matrix whose columns are the original vectors. Show your code, show the answer you get, and explain.

Problem 2: Write a procedure \text{basis(vlist)} with the following spec:

- \textit{input:} a list \textit{vlist} of Vecs
- \textit{output:} a list of linearly independent Vecs that span the same space as \textit{vlist}

The Vecs returned should be elements of \text{orthogonalize(vlist)}.

Your procedure should use the procedure \text{orthogonalize} defined in the provided module \text{orthog} but should call no other procedures. Ideally, it should be a one-line procedure.

When given the Vecs corresponding to
\[
[2, 4, 3, 5, 0], [4, -2, -5, 4, 0], [-8, 14, 21, -2, 0], \\
[-1, -4, -4, 0, 0], [-2, -18, -19, -6, 0], [5, -3, 1, -5, 2]
\]
the procedure might return Vecs that approximately correspond to

\[
[2, 4, 3, 5, 0], [3.81, -2.37, -5.28, 3.54, 0], \\
[-1.58, -0.73, 0.0009, 1.21, 0], [0.35, -3.16, 1.01, -0.99, 2]
\]

Note: In this problem and the next, to test whether a vector \( v \) should be considered a zero vector, you can see if the square of its norm is very small, e.g. less than \( 10^{-20} \).

**Problem 3:** Write a procedure `subset_basis(vlist)` with the following spec:

- **input:** a list `vlist` of vectors
- **output:** a list of linearly independent vectors that span the same space as `vlist` and that are in `vlist`

Your procedure should use `orthogonalize(vlist)` and no other procedure. Ideally, it should be a one-line procedure.

When given the Vecs corresponding to

\[
[2, 4, 3, 5, 0], [4, -2, -5, 4, 0], [-8, 14, 21, -2, 0], \\
[-1, -4, -4, 0, 0], [-2, -18, -19, -6, 0], [5, -3, 1, -5, 2]
\]

the procedure should return the Vecs corresponding to

\[
[2, 4, 3, 5, 0], [4, -2, -5, 4, 0], [-1, -4, -4, 0, 0], [5, -3, 1, -5, 2]
\]