Problem 1: Define datapoints \( p_1 = [10, 30], p_2 = [10, 10], p_3 = [30, 20] \). Compute the sum of squared distances from the datapoints to \([20, 15]\). Then compute the centroid of the datapoints, and compute the sum of squared distances from the datapoints to the centroid.

Problem 2: Let \( b = [1, 1/2], v_1 = [2, 2], v_2 = [2, 0] \).

1. Compute \( b \parallel v_1 \) and \( b \parallel v_2 \).

2. Let \( \hat{b} \) be the sum of the projections \( b \parallel v_1 \) and \( b \parallel v_2 \). We want to know if \( b - \hat{b} \) is orthogonal to \( v_1 \).
   Consider the following equations
   \[
   \langle b - \hat{b}, v_1 \rangle = \langle b, v_1 \rangle - \langle \hat{b}, v_1 \rangle \\
   = \langle b, v_1 \rangle - \langle \left( b \parallel v_1 + b \parallel v_2 \right), v_1 \rangle \\
   = \langle b, v_1 \rangle - \langle b \parallel v_1, v_1 \rangle - \langle b \parallel v_2, v_1 \rangle
   \]
   Calculate the value of the final expression on the right-hand side, \( \langle b, v_1 \rangle - \langle \left( b \parallel v_1 + b \parallel v_2 \right), v_1 \rangle \), by substituting \([1, 1/2]\) for \( b \) and \([2, 2]\) for \( v_1 \) and the vectors you computed for \( b \parallel v_1 \) and \( b \parallel v_2 \), and check if the result is zero.

3. We also want to know if \( b - \hat{b} \) is orthogonal to \( v_2 \).
   (a) Write down the above equations, suitably modified to compute the inner product of \( b - \hat{b} \) with \( v_2 \). The final expression should be
   \[
   \langle b, v_2 \rangle - \langle b \parallel v_1, v_2 \rangle - \langle b \parallel v_2, v_2 \rangle
   \]
   (b) Perform the analogous substitutions into this expression, and calculate the value, and check if it is zero.

Problem 3: Repeat the above problem but with one change: \( v_2 = [2, -2] \).
Problem 4: See the lecture slides. I show that

\[
\langle b - \hat{b}, v_1 \rangle = \langle b, v_1 \rangle - \langle \hat{b}, v_1 \rangle
\]
\[
= \langle b, v_1 \rangle - \langle \sigma_1 v_1 + \sigma_2 v_2 + \cdots + \sigma_n v_n, v_1 \rangle
\]

Ignoring the cross-terms, show using algebra that \( \langle b, v_1 \rangle = \langle \sigma v_1, v_1 \rangle \) by using the definition of \( \sigma_1 \).