Do the following problems, then do Problem 3 in `hw10-18.pdf` provided earlier. The following problems use the stencil `exchange_lemma_practice.py`. The third problem uses the stencil `The_Basis_other_problems.py`

**Exchange Lemma for vectors over \( \mathbb{R} \)**

**Problem 1:** Let \( S = \{[1, 0, 0, 0, 0], [0, 1, 0, 0, 0], [0, 0, 1, 0, 0], [0, 0, 0, 1, 0], [0, 0, 0, 0, 1]\} \), and let \( A = \{[1, 0, 0, 0, 0], [0, 1, 0, 0, 0]\} \). For each of the following vectors \( z \), find a vector \( w \) in \( S - A \) such that \( \text{Span } S = \text{Span } (S \cup \{z\} - \{w\}) \).

(a) \( z = [1,1,1,1] \)
(b) \( z = [0,1,0,0] \)
(c) \( z = [1,0,1,0,1] \)

**Exchange Lemma for vectors over \( GF(2) \)**

**Problem 2:** We refer in this problem to the vectors over \( GF(2) \) specified in Problem 5.14.4.

Let \( S = \{v_1, v_2, v_3, v_4\} \). Each of the following parts specifies a subset \( A \) of \( S \) and a vector \( z \) such that \( A \cup \{z\} \) is linearly independent. For each part, specify a vector \( w \) in \( S - A \) such that \( \text{Span } S = \text{Span } (S \cup \{z\} - \{w\}) \). (Hint: Drawing subgraphs of the graph will help.)

(a) \( A = \{v_1, v_4\} \) and \( z \) is \[
\begin{array}{cccccccc}
  a & b & c & d & e & f & g & h \\
  1 & 1 & & & & & & \\
\end{array}
\]

(b) \( A = \{v_2, v_3\} \) and \( z \) is \[
\begin{array}{cccccccc}
  a & b & c & d & e & f & g & h \\
  1 & 1 & & & & & & \\
\end{array}
\]

(c) \( A = \{v_2, v_3\} \) and \( z \) is \[
\begin{array}{cccccccc}
  a & b & c & d & e & f & g & h \\
  1 & & & & & & & 1 \\
\end{array}
\]

Finally, remember to do Problem 3.