Lecture 4

1. NFA to DFA conversion
2. Regular expressions
3. DFA to reg. ex. conversion

Theorem: Let $N = (Q_n, \Sigma, S_n, q_0, F_n)$ be an NFA. Then there exists a DFA $M = (Q_m, \Sigma, S_m, q_0, F_m)$ such that $L(M) = L(N)$.

Example NFA:

$$\begin{array}{c}
q_0 \\
\epsilon \\
q_1 \\
\epsilon \\
q_2 \\
\end{array}$$

Input: 00111

Time 0: $\{q_0\}$

Time 1: $\{q_0, q_1\}$

Time 2: $\{q_0, q_1, q_3\}$

Time 3: $\{q_0, q_1, q_3\}$

Time 4: $\{q_0, q_1, q_2, q_3\}$

Time 5: $\{q_0, q_1, q_2, q_3\}$

Accept because $q_2$ is possible at times and it is accepting.
Let $M$ be as follows:

states of $M$: $Q_M = \{ q_i \mid 1 \leq i \leq Q_N \}$

$S$ is the same as $N$

$S_M$ is defined as

\[ S_M(q, a) = \{ q' \mid \exists q' \in S_N(q, a) \} \]

$\text{for some } q \in S$

\[ S_M(q, a) = \bigcup_{q \in S_N(q, a)} S_M(q, a) \]

$F_M = \{ r \mid r \cap F_N \neq \emptyset \}$

where $E(S) = \{ q \in Q \text{ that are reachable from } q \}'s$ via $\delta$-transition
For ε-transitions:

\[ S_M(r, a) = \{ q' \mid \exists q \in \mathcal{Q} \text{ s.t. } \]

\[ q_{in} \in E(r), q_{out} \in E(S_M(q_{in})) \]

Reg. Languages are closed under reg. operations

U, \circ, *

\[ R = \epsilon(100)^* \circ U \epsilon(100)^* \]

\[ R_1 \quad R_2 \]

\[ R_1 = \epsilon(100)^* \]

\[ R_3 \quad R_4 \]

A regular expression (regex) over \( \Sigma \) is inductively defined:

R is a regex of size 1 if:

1. \( R = a \), \( a \in \Sigma \)
2. \( R = \Sigma \)
3. \( R = \emptyset \)
$R$ is a regex of size $n > 1$ if for some regexes $R_1$, $R_2$

4. $R = R_1 \cup R_2 \}$ and $\text{size}(R_1) + \text{size}(R_2) + 1 = n$

5. $R = (R_1 \circ R_2$

6. $R = (R_1)^*$ and $\text{size}(R_1) + 1 = n$

If $R$ is a regex then $L(R)$ is

1. $L(R) = \{a, b\}$

2. $L(R) = \{c, e\}$

3. $L(R) = \emptyset$

4. $L(R) = L(R_1) \cup L(R_2)$

5. $L(R) = L(R_1) \circ L(R_2)$

6. $L(R) = (L(R))^*$

Then: if $R$ is a regex then $L(R)$ is regular

order of operation: $\ast$, concat, $\cup$
Def a GNFA consists of $Q$, $\Sigma$, $\delta$, $\lambda$, $q_0$, $q_{\text{accept}}$

s.t. $Q$: set of states

$\Sigma$:

$\lambda: Q \setminus \{q_{\text{accept}}\} \times Q \setminus \{q_{\text{start}}\}$

$\rightarrow$ regular expressions over $\Sigma$

$w$ is accepted by $G$ if $\exists$ sequence $q_1, q_2, \ldots, q_n$

s.t. $w = w_0, \ldots, w_n$ s.t. $q_0 = q_{\text{start}}$, $q_n = q_{\text{accept}}$

and $w \in \bigcup_{i \leq n} \ell(q_i, q_{i+1}) \quad \forall 0 \leq i \leq n$
removing $a_2$ from the GNFA

\[ q_{\text{start}} \rightarrow q_0 \rightarrow q_2 \rightarrow q_\text{Accept} \]

in general

\[ q_1 \rightarrow q_2 \rightarrow q_3 \rightarrow q_4 \]

becomes: