HW3

Due: October 1, 2015

Attach a fully filled-in cover sheet to the front of your printed homework. Your name should not appear anywhere except on the cover sheet; each individual page of the homework should include your banner ID instead.

While collaboration is encouraged in this class, please remember not to take away notes from any collaboration sessions. Also please list the names and logins of any of your collaborators at the beginning of each homework.

Please monitor Piazza, as we will post clarifications of questions there. You should hand in your solutions by 12:55 to the CSCI 51 bin on the second floor of the CIT. Late homeworks are not accepted.

Problem 1

Let $L$ be the language denoted by the regular expression $(01 \cup 10)^*111$.

a. Provide an NFA whose language is $L$.

b. Provide a regular expression for the language $L^c$. (Show your work.)

Problem 2

Let $B$ be a language, and $A$ and $C$ be regular languages over $\Sigma = \{0, 1\}$, with $A \subseteq B \subseteq C$. Which of the following statements is true? Prove your answer.

(i) $B$ must be regular.

(ii) $B$ is regular in some cases, and not regular in some cases.

(iii) $B$ must not be regular.
Problem 3

For each of the following languages using the alphabet \{0, 1\}, either find a regular expression for the language, or prove that it is not regular.

a. \(L_a = \{0^n \mid n \text{ is a power of two}\}\).

b. \(L_b = \{0^n1^n \mid 0 \leq n \leq 100\}\).

c. \(L_c = \{w \mid w \text{ has an even total number of 1s}\}\).

d. \(L_d = \{xx \mid x \text{ is any binary string}\}\).

The following questions are lab problems.

Lab Problem 1

The Enterprise are exploring new life and new civilizations on planets near the edge of the galaxy when accidentally find themselves well into the Neutral Zone.\(^1\) Unsurprisingly, they attract the attention of a nearby Klingon vessel. In order to prevent the Klingons from calling in with a fleet of Warbirds to act out a real-life Kobayashi Maru scenario\(^2\) on the Enterprise and its crew, Uhura is jamming the Klingons’ scanners. They may only un-jam their communications devices if they can recognize the following language on their malfunctioning computer, which now only works as a DFA. Will the Enterprise be successful in escaping before the Klingons can call in back-up? (You do not have to answer that.)

Use the pumping lemma to show that

\[L = \{0^i1^j \mid i > j\}\]

is not regular.

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\(^1\)Due to perpetual hostility between the Klingons and the Federation, no party is allowed to enter the Neutral Zone

\(^2\)The Kobayashi Maru is a notoriously hard training exercise testing how Star Fleet cadets deal with a “no-win scenario”, i.e. certain death
Lab Problem 2

Recall that a \textit{pumping length} for a language \( A \) is a positive integer \( p \) such that all strings \( s \in A \) with \( |s| \geq p \) can be written in the form \( xyz \), where

(i) \( |xy| \leq p \),
(ii) \( |y| \geq 1 \),
(iii) and \( xy^iz \in A \) for all \( i \geq 0 \).

Also, recall from class that if \( A \) is finite with \( \ell \) being the length of \( A \)'s longest string, \( p = \ell + 1 \) is a valid pumping length for \( A \), because there are no strings \( s \in A \) with \( |s| \geq \ell + 1 \), which makes it vacuously true that all such strings satisfy the three conditions above.

The pumping lemma states that every regular language has a pumping length.

The \textit{minimum pumping length} of a language \( A \), \( p_{\text{min}} \), is the smallest pumping length for \( A \). Note that this implies every integer \( p \geq p_{\text{min}} \) is also a valid pumping length for \( A \).

For example, if \( A = ab^* \) the minimum pumping length is two. To justify this, note that the string \( s = a \) is in \( A \) yet cannot be pumped at all; writing it as \( xyz \) we must have \( x = \epsilon, y = a, z = \epsilon \), and then \( xz \) is not in \( A \). So 1 is not a pumping length. But 2 is a pumping length, because for any string \( |s| \geq 2 \) we can take \( x = a, y = b, \) and \( z \) to be everything else, and we have that \( |xy| \leq 2, |y| \geq 1, \) and \( xy^iz \in A \) for all \( i \geq 0 \).

For each of the following languages, give the minimum pumping length \( p_{\text{min}} \) and prove your answer.

a. \( bb^* \)
b. \( ba(bb^*a)^*a \)
c. \( L = \{ w \in \{a, b\}^* \text{ such that } w \text{ ends in } ab \} \)
d. \( L = \{ aab, bba, b \} \)
e. \( L = \{ w \in \{a, b\}^* \text{ such that } w \text{ does not end in } ab \} \)
Lab Problem 3

Consider the language $F$ defined over the alphabet $\Sigma = \{a, b, c\}$ as

$$F = \{a^i v : i \geq 0, \text{ } v \text{ is a sequence of } b\text{'s and } c\text{'s, and if } i = 1 \text{ then } v \text{ is palindromic: it is the same read backwards as forwards.}\}$$

a. Show that $F$ is not regular.

b. Give a pumping length $p$ and demonstrate that, for all strings $w \in F$ such that $|w| \geq p$, we can write $w = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in F$ for all $i \geq 0$. In other words, you can see that the pumping lemma is not helpful for proving $F$ is not regular.

c. Explain why parts (a) and (b) do not contradict the pumping lemma.