Logical Equivalences

Given booleans $p$, $q$, and $r$, the following logical equivalences hold:

1. **Commutative Laws:**
   
   \[
   p \& q == q \& p \\
   p \mid q == q \mid p
   \]

2. **Associative Laws:**
   
   \[
   (p \& q) \& r == p \& (q \& r) \\
   (p \mid q) \mid r == p \mid (q \mid r)
   \]

3. **Distributive Laws:**
   
   \[
   p \& (q \mid r) == (p \& q) \mid (p \& r) \\
   p \mid (q \& r) == (p \mid q) \& (p \mid r)
   \]

4. **Identity Laws:**
   
   \[
   p \& -1 == p \\
   p \mid 0 == p
   \]

5. **Negation Laws:**
   
   \[
   p \& (~p) == 0 \\
   p \mid (~p) == -1
   \]
6 Double Negate Law:

\[ \sim(\sim p) = p \]

7 Idempotent Laws:

\[ p \& p = p \]

\[ p \lor p = p \]

8 Universal Bound Laws:

\[ p \& 0 = 0 \]

\[ p \lor -1 = -1 \]

9 De Morgan’s Laws:

\[ \sim(p \& q) = \sim p \lor \sim q \]

\[ \sim(p \lor q) = \sim p \& \sim q \]

10 Absorption Laws:

\[ p \lor (p \& q) = p \]

\[ p \& (p \lor q) = p \]

11 Logical Negation of 0 and not 0:

\[ !<\text{non-zero}> = 0 \]

\[ !0 = 1 \]