Counting

number of zero/one strings of length \( n = 2^n \)

Product Rule
\( S_1, S_2, ..., S_k \) then \( |S_1 \times S_2 \times ... \times S_k| = |S_1| \times |S_2| \times ... \times |S_k| \)

Proof: Easy Induction

Count functions
\( |X| = n \) and \( |Y| = m \)
So number of functions \( f : X \rightarrow Y = m^n \)

Ex \( f : X \rightarrow \{0, 1\} \) we get \( 2^n \)
Ex \( f : X \rightarrow \{0, 1, 3\} \) we get \( 3^n \)

- number of functions \( f : X \rightarrow Y \) that are injective (one-to-one) =
  \[ m(m-1)...(m-n+1) \]

- number of surjective function (onto) is complicated

- number of bijective functions the same as number of injective functions

**Prop** if \( |X| = |Y| \) and \( f : X \rightarrow Y \) is injective then \( f \) is surjective.

Thus we can just count the number of bijections is the number of injections if the size of the sets is the same.

Factorial

Definition of ‘n factorial’: For \( n \in \mathbb{Z} \ n \geq 1 \)

\[ n! = n(n-1)...(1) \]

define \( 0! = 1 \)

Permutation

Consider \( f : X \rightarrow X \). If \( f \) is a bijection then, it is called a **permutation**. Let \( |X| = n \). The number of permutations of \( X \) is \( n! \)
Definition for $n \geq k \geq 0 \in \mathbb{Z}$

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

This is read 'n choose k'

Claim: $\binom{n}{k}$ = number of ways to select $k$ objects from an $n$ element set
also number of subsets of size $k$ of an $n$-element set

Note that number of subsets of $n$-element sets = $2^n$

$\binom{n}{k}$ = number of 0/1 strings of length $n$ with exactly $k$ 1’s