Class Notes

Counting

number of zero/one strings of length $n = 2^n$

Product Rule

$S_1, S_2, \ldots, S_k$ then $|S_1 \times S_2 \times \ldots \times S_k| = |S_1| \times |S_2| \times \ldots \times |S_k|$

Proof: Easy Induction

Count functions

$|X| = n$ and $|Y| = m$

So number of functions $f : X \to Y = m^n$

Ex $f : X \to \{0, 1\}$ we get $2^n$

Ex $f : X \to \{0, 1, 3\}$ we get $3^n$

- number of functions $f : X \to Y$ that are injective (one-to-one) =

  $$m(m - 1)(m - n + 1)$$

- number of surjective function (onto) is complicated

- number of bijective functions the same as number of injective functions

  Prop if $|X| = |Y|$ and $f : X \to Y$ is injective then $f$ is surjective.

  Thus we can just count the number of bijections is the number of injections if the size of the sets is the same.

Factorial

Definition of ‘$n$ factorial’: For $n \in \mathbb{Z}$ $n \geq 1$

$$n! = n(n - 1)(1)$$

define $0! = 1$

Permutation

Consider $f : X \to X$. If $f$ is a bijection then, it is called a permutation. Let $|X| = n$. The number of permutations of $X$ is $n!$
Definition for $n \geq k \geq 0 \in \mathbb{Z}$

\[ \binom{n}{k} = \frac{n!}{(n-k)!(k)!} \]

This is read ‘$n$ choose $k$’

Claim: $\binom{n}{k} =$ number of ways to select $k$ objects from an $n$ element set
also number of subsets of size $k$ of an $n$-element set

Note that number of subsets of $n$-element sets = $2^n$

$\binom{n}{k} =$ number of 0/1 strings of length $n$ with exactly $k$ 1’s