Counting with Bijections

*Notation:* \( \{0, 1\}^n \) denotes the set of 0/1 strings of length \( n \).

A *bijective proof* proves that the cardinalities of two finite sets \( A \) and \( B \) are equal by constructing a bijection from \( A \) to \( B \).

A *counting function* is a function whose domain is \( \mathbb{Z} \) or \( \mathbb{N} \) and whose codomain is \( \mathbb{Z} \) or \( \mathbb{N} \). The input to a counting function is a parameter of some type of structure we’d like to count, and the output of a counting function is the number of such structures.

**Induction**

*Induction* is a proof method to prove the proposition \( P(n) \) is true \( \forall n \in \mathbb{N} \) (or \( \forall n \geq b \) for some \( b \in \mathbb{N} \).) A proof by induction comprises two parts:

**Base Case** Prove \( P(b) \) is true (where \( b \) is the smallest index.)

**Induction Step** Assume \( P(k) \) is true for some fixed but arbitrary \( k \) (this is called the *inductive hypothesis*), and then show that \( P(k) \implies P(k + 1) \).

After these two steps, conclude what you’ve done to finish off the proof.

*Note:* If you use a predicate \( P(n) \) in your proof, you **must clearly define** what \( P(n) \) is.