Sets

**Definition:** The *empty set*, denoted {} or ∅, is the set with no elements.

Some set properties:

- Any set is a subset of itself (also called an *improper subset.*)
- The empty set is a subset of any set.
- |P(S)| = 2^{|S|}.

The *set element method* is a way to show that two sets $A$ and $B$ are equal:

1. Show $A \subseteq B$ by considering an arbitrary element $a \in A$, and showing that $a \in B$.
2. Show $B \subseteq A$ by considering an arbitrary element $b \in B$, and showing that $b \in A$.
3. Conclude that $A = B$.

**Proposition (Distributive Laws):**

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

**Quantifiers**

- $\forall$ is the *universal quantifier*, and means “for all.”
- $\exists$ is the *existential quantifier*, and means “there exists.”

**Proofs**

An *existential proof* proves the existence of something.

A *constructive proof* proves an existential claim by constructing the object whose existence is in question.

A *proof by cases* is a proof that examines an exhaustive set of cases, and proves that the claim holds in every case.

**Proposition:** $\exists x, y \in \mathbb{R}$, where $x, y \not\in \mathbb{Q}$, such that $x^y \in \mathbb{Q}$. 