Functions

Given \( f : A \rightarrow B \)

- \( A \) is known as the Domain of the function
- \( B \) is known as the Co-Domain of the function
- Image/Range = \( \{ b \in B | \exists a \in A \text{ s.t. } f(a) = b \} \)

Injectivity, Surjectivity, Bijectivity

Defn: \( f \) is surjective (onto) if \( \forall b \in B \exists a \in A \text{ s.t. } f(a) = b \)

Defn: \( f \) is injective (one-to-one) if \( \forall b \in B \exists \text{ at most one } a \in A \text{ s.t. } f(a) = b \)

Defn: \( f \) is bijective if it is both injective and surjective

\( f : A \rightarrow B \) (bijective)

- \( f \) is injective \( \implies \) \( |B| \geq |A| \)
- \( f \) is surjective \( \implies \) \( |B| \leq |A| \)
- \( f \) is bijective \( \implies \) \( |B| = |A| \)

Also if we have two sets \( A \) and \( B \) that are the same size, then there exists a bijection between these two sets (Note that not all functions between these two sets are necessarily bijections)

Donuts

Set up: You are going to PVDonuts and want to buy 10 donuts. There are 4 different types of donuts you can buy.
Claim: The number of different combos of donuts you can buy is equal to the number of 0/1 strings of length 13 with exactly three 1’s.