Equivalence Class

**Defn:** $R$ on $A$, $[a]_R = \{x \in A | (x, a) \in R\}$. $[a]_R$ is the equivalence class of $a$.

Partitions

**Defn:** A partition of set $A$ is a collection of disjoint subsets of $A$ that cover $A$. So, $P = B_1B_2..B_n$ where $B_i \subseteq A$

- $\forall a \in A, a \in B$ for some $i$
- $\forall i, j B_i \cap B_j = \emptyset$

**Prop:** Given an eq relation on $R$ on a set $A$, the eq classes partition $A$.

*Proof.* Let $R$ be eq relation on $A$. Consider all distinct equiv classes. (distinct = not equal as a set).

1. $\forall a \in A, (a, a) \in R \implies a \in [a]_R$ (bc Reflex)

2. Let $B_1, B_2$ be distinct so $B_1 \neq B_2$
   
   Claim: $B_1 \cap B_2 = \emptyset$
   
   Assume for the sake of contradiction that $B_1 \cap B_2 \neq \emptyset$
   
   Let $x \in B_1 \cap B_2$
   
   $B_1 = [y]_R$ and $B_2 = [z]_R$
   
   Let $w \in B_1 \implies (w, y) \in R$
   
   $x \in B_1 \implies (x, y) \in R \implies (y, x) \in R \implies (w, x) \in R$
   
   $x \in B_2 \implies (x, z) \in R \implies (w, z) \in R \implies w \in B_2$
   
   So $B_1 \subseteq B_2$