Set Equivalence

The following are two sample proofs of the equivalence

$$(A \cap B) \cup (A - B) = A \cap (B \cup (A - B)).$$

One uses the element method. The other uses set algebra.

**NOTE:** Drawing a Venn diagram of each does not constitute a proof and will not be graded as such.

**Element Method**

**Requirements**

1. Show the left side of the of the equation is a subset of the right.
2. Show the right side of the of the equation is a subset of the left.
3. Conclude they are equal.

**Proof**

Suppose $x \in (A \cap B) \cup (A - B)$. Then either $x$ is in both $A$ and $B$ or $x$ is in $A$ but not $B$. Either way, $x$ must be in $A$. Therefore, $x$ must either be in the portion of $A$ that does not overlap with $B$ or the portion that does. So either $x \in A - B$ or $x \in B$. But we also know $x$ is definitely in $A$, so $x \in A \cap (B \cup (A - B))$. And since $x$ was arbitrary, this is true for all elements in the set, and therefore the set as a whole. This proves the first direction. For the second, suppose $x \in A \cap (B \cup (A - B))$. Then $x \in A$ and $x \in (B \cup (A - B))$. Therefore either $x$ is in $B$ or $x$ is in $A$ but not $B$. Since $x$ is also in $A$, $x$ is either in $(A \cap B)$ or $(A - B)$. So $x \in (A \cap B) \cup (A - B)$. This proves the second direction, and as both directions hold, the equality is proven.

**Set Algebra**

**Requirements**

1. Conversion of one side of the equation to the other (or conversion of both sides to an identical expression) using *stated* laws of set algebra
2. Conclusion based on the biconditionality of the steps taken
Set Equivalence

Proof

\[(A \cap B) \cup (A - B)\]  
\[(A \cap B) \cup (A \cap B^c)\]  (Set Difference Law)  
\[A \cap (B \cup B^c)\]  (Distribution)  
\[A \cap U\]  (Complement Law)  
\[A\]  (Identity Law)  
\[A \cap (A \cup B)\]  (Absorption)  
\[A \cap (B \cup A)\]  (Commutivity)  
\[A \cap ((B \cup A) \cap U)\]  (Identity Law)  
\[A \cap ((B \cup A) \cap (B \cup B^c))\]  (Complement Law)  
\[A \cap (B \cup (A \cap B^c))\]  (Distribution)

All these steps are biconditionally true, therefore the equality holds.