The Product Rule

**TASK:** Suppose we go to the sandwich shop, and we want to order a combo special. The combo special includes a sandwich, and side, and a drink.

There are 5 different kinds of sandwiches, 6 different kinds of sides, and 8 different kinds of drinks. How many different ways could we order a combo special, and why? Use a Cartesian product in your argument.

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True Or False

**TASK:** For each of the following statements, determine if it’s true or false. If it’s false, modify the statement to make it true.

i. For all \( n > 0 \), there are more relations on a set \( A \) of size \( n \) than there are functions from \( A \) to \( A \).
ii. For all \( n > 0 \), there are infinitely many functions from an infinite set to a set of size \( n \).

iii. For all \( n > 1 \), there are infinitely many functions from an infinite set to a set of size \( n \).

iv. There are infinitely many functions from a finite set to an infinite set.

When we first talked about functions, these functions only took in one input. Since then, we’ve seen propositions, which are functions that can take in more than one input! In general, functions can take in one or more inputs.

Note that if the function takes in more than one input, say \( n \) inputs, we can think of it taking in one input, where each input is a tuple of length \( n \).

**TASK:** Now, determine whether the statements below are true or false, and like before, if they’re false, modify them to make them true.

i. Consider the number functions of \( n \) inputs. We only need to know how many values each input can take on to determine this number.

ii. Consider a function of 3 inputs, where each input can take on 0, 1, or 2. If we think about this function as a function of one input, there are \( 3^3 \) possible inputs to it.
iii. Consider functions of 2 inputs, where each input can take on either 0 or 1, and the function must output to 0 or 1. There are $2^2$ such functions.

iv. Consider functions of 2 inputs, where each input can take on 0, 1, or 2, and the function must output to 0 or 1. There are $3^{(3^2)}$ such functions.

v. Consider functions of 2 inputs, where each input can take on either 0 or 1 and outputs either 0 or 1, but the order of the inputs does not affect that output of the function. For example, $f(x, y)$ could be $(x \land y)$. There are only 2 possible such functions.

Checkpoint - Call over a TA

Permutations and Combinations

A permutation on a set $A$ of size $n$ is a bijection from $A$ to $A$. Think about laying out the elements of $A$ in an ordered list. Then, our permutation gives each element in $A$ a (potentially) new spot in the list (the reason we write potentially is because the element could be mapped to itself).
**TASK:** Explain why there are $n!$ different bijections from a set of size $n$ to $n$. That is, explain why there are $n!$ different permutations of a set of size $n$.

**TASK:** Suppose we want to know the number of subsets of size $k$ of a set of size $n$. What is the general formula for this, and why?

**Sort**

Sort the following in order from largest to smallest.

i. The number of permutations of all the letters in the alphabet.

ii. The number of subsets of size 6 of the letters of the English alphabet (one such subset is $\{a, b, c, d, e, f\}$).

iii. The number of 6 letter words made of unique letters (one such 6-letter word is “abcdef”).

iv. The number of (any sized) subsets of the set of the letters in the English alphabet.

v. The number of subsets of size 20 of the letters of the alphabet.
Counting Arguments

A counting argument shows that the LHS (lefthand side) and the RHS (righthand side) of some equation count the same thing. Instead of using algebraic manipulation, we explain why both sides enumerate the elements of some set, just in different ways. That is, we think about a set, and we come up with a story that corresponds to the LHS that describes an enumeration of the elements of that set, and then we come up with a different story that corresponds to the RHS that describes an enumeration of the elements of that very same set.

As computer scientists, counting arguments are particularly important to us. One reason for this is because we can use them to prove the equivalence of two algorithms that count things.

**TASK:** Explain the difference between showing a bijection exists between two sets and a counting argument as described above.

**TASK:** Use a counting argument to prove the following identity:

\[ \binom{n}{k} = \binom{n}{n-k} \]
**TASK:** Use a counting argument to prove the following identity:

\[ \sum_{k=0}^{n} \binom{n}{k} = 2^n \]