These are solutions for the practice final. Please note that these solutions are intended to provide you with a basis to check your own answers. In order to get full credit on the exam, you must show all work and fully explain your answers. These solutions do not necessarily provide a full explanation. If you are confused about these answers, make sure to review the lecture slides or come to TA hours.

Problem 1

Solution:

1. $O(\log n)$, because at worst you will only need to downheap the height of the heap.

2. $O(n^2 \log n)$, because Kruskal’s algorithm sorts the edges which requires $O(|E| \log |E|)$, which equals $O(n^2 \log n^2)$, which simplifies to $O(n^2 \log n)$ by the properties of logarithms.

3. $O(n)$, because you may need to iterate through the entire linked list.

4. $O(n)$, using radix sort.

5. $O(n \log n)$. You could do a breadth-first search starting from a random node and decorate each node as it is added to the tree. This visits every node and edge, so it is $O(|V| + |E|)$, or $O(n \log n)$

6. Since each node is visited once in a pre-order traversal, the runtime is $O(n)$.

CS16 — Introduction to Algorithms and Data Structures  Spring 2017
Problem 2
Solution:
Using what we know about the order of binary search trees we can avoid searching sections of the
tree that aren’t within the \( k_1, k_2 \) range. If a node is less than our \( k_1 \) key then we know all nodes to
the left of this node will also be smaller and therefore don’t need to be visited. Also, if a node value
is higher than \( k_2 \) we know we don’t have to visit any children to the right of that node. Using this
we can traverse the tree with a modified in-order traversal in which we only visit the left child if the
current node is greater than \( k_1 \) and only visit the right child if the current node is less than \( k_2 \).

```python
function traverseRange(tree, k1, k2)
    """ Transforms: Tree, int, int -> array of TreeNodes
    Purpose: calls the traversalHelper to find and
    add nodes with keys in a given range (k1<k2) to
    the 'results' array
    """
    result = []
    if root is not null:
        traversalHelper(tree.root(), k1, k2, result)
    return result

function traversalHelper(current, k1, k2, result)
    """ Transforms: TreeNode, int, int, array -> nothing
    Purpose: checks left and right children for range,
    traversing the graph in modified in-order.
    'result' may be modified
    """
    leftChild = current.leftChild()
    rightChild = current.rightChild()
    if leftChild is not null and current.key > k1:
        traversalHelper(leftChild, k1, k2, result)
    if current.key >= k1 and current.key <= k2:
        result.add(current.key)
    if rightChild is not null and current.key < k2:
        traversalHelper(rightChild, k1, k2, result)
```

This algorithm runs in \( O(h + s) \) because you may need to travel down the entire height \( h \) of
the tree, and you must visit every node \( s \) between \( k_1 \) and \( k_2 \) exactly once.
Problem 3  

**Solution:**

We can simply run Kruskal’s algorithm, but add the edges in $F$ into our spanning tree before running Kruskal’s algorithm. This way, all edges in $F$ will be in our final tree $T$, and we are still adding all possible edges of minimum weight because Kruskal’s will sort the remaining edges.

```python
function modifiedKruskal(G):
    """ Transforms: graph G -> array ST
        Purpose: finds the lightest spanning tree such that all edges in F are included. Returns array of the edges in the spanning tree."
    ""
    for vertices v in G:
        makeCloud(v)
    ST=[] //spanning tree
    for all edges (u,v) in G where inF(e) is true:
        add(u,v) to ST
        merge clouds containing u and v
    for all edges (u,v) in G where inF(e) is false, sorted by weight:
        if u and v are not in same cloud:
            add (u,v) to ST
            merge clouds
    return ST
```

This algorithm is $O(|E| \log |E|)$ because the slowest operation is sorting the edges not in $F$, which is $O(|E| \log |E|)$.

Alternatively, you could set all the edges in $F$ to $-\infty$ and run Kruskal’s normally, since would automatically choose them first.
Problem 4

Solution:
Perform a modified topsort on the graph. Set the cost of every node to \(-\infty\) and the start node to 0. Push all sources onto the stack. While the stack is not empty, pop a node and examine all of its neighboring nodes. If the cost of the neighbor is less than the that of the node plus the weight of the edge between them, update the cost of the neighbor to this higher value. Delete the edge between the node and its neighbor. If the neighbor is now a source, push it on to the stack. Once the target node is popped from the stack, we know that its cost will not change since it must be a source to have been pushed on the stack. Thus, we can immediately return the target node’s cost.

The pseudocode looks something like this:

```python
function longestPath(G, s, t):
    """ Transforms: graph G, start node s, target node t ->
    length of longest path between s and t
    """
    stack = Stack()
    for v in G.vertices():
        v.cost = -infinity
        if v is source:
            stack.push(v)
    s.cost = 0
    while stack not empty:
        v = stack.pop()
        if v == t:
            return v.cost
        for each edge (v, w):
            if w.cost < v.cost + weight(v, w):
                w.cost = v.cost + weight(v, w)
                delete edge (v, w)
                if w is source:
                    stack.push(w)
    return infinity
```
Problem 5

Solution:
Runtime for Algorithm A:
\[ T(|V|) = T(|V|/2) + O(|V|^2) \]
Recall the master theorem: \( a = 1, b = 2, d = 2 \)
so \( T(|V|) \) is \( O(|V|^2) \)

(remember, we’ll give you the master theorem for the actual exam if you need it!)

Runtime for Algorithm B:
\( O(|E| \log(|E|)) \) to sort the edges using merge or quick sort

If the graph is fully connected, algorithm B has a runtime of \( |V|^2 \log(|V|^2) \), which simplifies to \( O(|V|^2 \log(|V|)) \). So Algorithm A is faster in this case. But if the graph has much fewer edges (suppose \( |E| \) is roughly equal to \( |V| \)), then \( O(|E| \log |E|) \) is much less than \( O(|V|^2) \), so algorithm B would be better in this case.
Problem 6

Solution:
This is another dynamic programming problem, where the best solution for weight \( k \) would be calculated from the known best solution of weight \( k - 1 \).

\[
\text{MaximizeKnapsack}(C, \text{items})
\]

```
### Transforms maximum weight C, array of items -> int weight
Purpose: calculates the max value that can be stored in
a knapsack that can hold a weight C.
###

maxVal = array of size C+1 //since counting starts at 0
maxVal[0] = 0
for weight = 1 to C:
    max = negative_infinity
    for item in items:
        if weight >= item.weight:
            value = maxVal[weight-item.weight] + item.val
            if value > max:
                max = value
        maxVal[weight] = max
return maxVal[C]
```
Problem 7

Solution:

```python
def reverseList(head):
    """ Transforms node head -> nothing
        Purpose: reverses a list of nodes
    """
    prev = null
    current = head
    next = null
    while(current != null):
        next = current.next
        current.next = prev
        prev = current
        current = next
    head = prev
```

This method iterates through the list and ‘reverses’ the next pointers. Try hand-simulating with a short example to see how this works and end up with a reversed list! Since the method goes over each element of the list once (from the while loop), the run-time is linear i.e. \( O(n) \)
Problem 8

Solution:
Here’s pseudocode for the sort function:

```python
def sort(S):
    """ Transforms unsorted stack of ints S -> sorted stack of ints R
    Purpose: sorts a stack of integers. """
    Stack R = new Stack()
    while !S.isEmpty():
        temp = S.pop()
        while(!R.isEmpty() and R.peek() > temp):
            S.push(R.pop())
        R.push(temp)
    return R
```

The idea is that each element of S is taken out and placed on the 'placeholder' stack R, only if something already on top of R isn’t smaller than the element we’re looking at. Try to hand-simulate on a small example!
Problem 9

Solution:
A trie is used in autocomplete. Each node represents a prefix. To perform autocomplete, the proper prefix is found by using the `find()` procedure, which successively scans the nodes at each level to find the proper branch for the prefix and returns the last node of the prefix in the trie. Then, `dfs()` is used to scan all of the branches of the tree rooted at the last node of the prefix to find all of the viable words and phrases beginning with that prefix.

In the functional paradigm, there is no notion of state. That is, each function depends only on the input data and reliably returns the same output for a given input. Nothing else in the program can affect what is going on inside of a function.

Another defining characteristic of the functional paradigm is that you can think of a program as a composition of functions.

One more item is that data in the functional paradigm is immutable. That is, once a variable in a function is assigned data, it can not be modified to hold different data. Because of this, we cannot use for loops in functional programming.

A* and Dijkstra’s both seek to find the shortest path from a start node to a goal node. A*’s heuristic may do so faster by examining paths in an order that allows you to find the goal faster rather than strictly increasing the lengths of the paths that are being examined as is done in Dijkstra’s. However, there is no guarantee that A* will be any faster than Dijkstra’s. After all, you can consider Dijkstra’s a special case of A* where the heuristic equals zero for every node, and thus in the worst-case they have the same big-O runtime.

P is the set of all search problems (having a well-defined solution state and checkable in polynomial time) that are also solvable in polynomial time. NP is a superset of P, and is simply the set of all search problems, regardless of whether they are solvable in polynomial time.

For offline algorithms, you have all of the data that you are going to use beforehand, so can optimize your solution based on that information. For online algorithms, you are continuously getting new data, and have to handle it in the moment without knowing what data will come next. There are many examples of offline algorithms that we used, such as any of the sorting methods. Expanding stacks and queues were an online algorithm, as was the incremental hull sort algorithm.