Gaussian Process Dynamical Models

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Foci

• Want low-dimensional representations of high-dimensional data
  – Find the latent space
  – Dimensionality reduction

• Want to model *dynamics* of the system
  – How it evolves over time
  – Not just distance in input space
Domain

• Human motion (gait)
  – 62 dimensions, video
  – Regular DR treats each frame (pose) separately
  – Want to include how poses relate over time
Non-linear Dynamical Model

- State evolution in latent space
  \[ x_t = f(x_{t-1}; A) + n_{x,t} \]
  \[ f(x; A) = \sum_i a_i \phi_i(x) \]
- Observation (latent->observed)
  \[ y_t = g(x_t; B) + n_{y,t} \]
  \[ g(x_t; B) = \sum_j b_j \psi_j(x) \]
- Model:
Issues

• Need parameters A and B and basis functions!
  – and enough data to constrain them
• Why not marginalize them out?
  – Can do it in closed form
Latent->observed mapping

• isotropic Gaussian prior on B, marginalize over g:

\[ p(Y | X, \bar{\beta}) = \frac{|W|^N}{\sqrt{(2\pi)^{ND} |K_Y|D}} \exp \left( -\frac{1}{2} \text{tr}(K_Y^{-1}YW^2Y^T) \right) \]

• Use RBF for kernel
Latent dynamics

- Use 1\textsuperscript{st} order Markov, marginalize over $A$ with isotropic Gaussian prior:

$$p(X|\alpha) = p(x_1) \frac{1}{\sqrt{(2\pi)^{(N-1)d} |K_x|^d}} \exp\left(-\frac{1}{2} \text{tr}(K_x^{-1}X_{out}X_{out}^T)\right)$$

- use linear + RBF kernel
Final Model

• All coordinates are jointly correlated, as are poses.

• Simple, no?
Learning

• All you have is Y, the observed variables (62 D human pose information)
  – minimize negative log-posterior numerically:
    \[ L = -\ln p(X, \alpha, \beta | Y) \]
  – Initialize latent coordinates with PCA
    • 3D, because 2D is unstable
Results

PCA

GPDM
New motion generation

Original

New
Other uses

• Forecasting
  – Use mean-prediction (motion generation) to look ahead

• Missing Data
  – Use mean-prediction to fill in gaps
    • Must increase uncertainty in training data via downsampling
Points / Issues

• Dimensionality of latent space is set via PCA space.
  – What if unknown?
• Scalability
  – Because of correlation between points, difficult to make sparse.
Want more?

- http://www.dgp.toronto.edu/~jmwang/gpdm