1. In this problem we investigate the space requirements of the DNA-based algorithm for solving the Hamiltonian Cycle problem discussed in Lecture 05.

   (a) Show that this algorithm may at some point have $\Omega(2^n)$ partial tours under consideration. (To do this, give an infinite family of graphs that exhibit this property.)

   (b) Give an upper bound on the number of partial tours of length $k$ that the algorithm will consider on an instance with $n$ vertices.

   (c) A kilogram of matter contains on the order of $10^{26}$ atoms, so optimistically one might be able to store say $10^{24}$ partial tours in a kilogram of matter. How big of an instance can you solve with:

      i. The 1 kg DNA computer in your lab
      ii. A DNA computer the size of the sun ($10^{25}$ kg)
      iii. A DNA computer the size of the universe ($10^{55}$ kg)

   (d) Many techniques are known to prune the search space in Hamiltonian Cycle problems (see CSCI 1490 and CSCI 2580 for details). Consider a classical computer that can consider $10^9$ operations tours per second and does enough pruning that it needs to consider only $10^{n/10}$ tours. How large of a graph can you consider with this classical computer in $10^4$ seconds (about 3 hours)?

2. In Lecture 06 a sketch is given of a key part of the reduction from a computation on a Turing machine to one by a tiling system. Complete the sketch of the reduction or explain in detail a proof found in other sources.