Optimization Algorithms for Planar Graphs, Homework for January 22, 2013

Rootward computations

Problem 1. Give pseudocode for a linear-time algorithm to find, for a given rooted tree with an assignment $w(\cdot)$ of nonnegative weights to nodes, a minimum-weight set $S$ such that each node of the tree is in $S$ or is a parent or child of $S$.

Separators

Lemma 1 (Tree edge separator of Vertex/Edge Weight). Let $T$ be a binary tree, and let $w(\cdot)$ be a $\frac{1}{3}$-proper assignment of weight to the vertices and edges such that degree-three vertices are assigned zero weight. There is a linear-time algorithm to find an edge $e$ such that every tree in $T - \{e\}$ has at most two-thirds of the weight.

Problem 2. Prove Lemma 1.

Edge-centric graphs

Problem 3. For the graph drawing above, give the edge-centric representation as a graph with edges $\{a, b, c, d, e\}$.

Using the graph, for each of the following categories, give an example of the category that doesn’t belong to the previous categories.

1. simple cycle of arcs
2. simple cycle of darts
3. simple directed path
4. simple path of edges
5. path of edges
6. directed walk
7. walk