Photon Mapping Homework

Due: 11:00am on Feb 26

All work must be your own. You may use lecture notes, Jensen’s paper, references, and the textbook. Please write legibly or type your answers. Justify all answers.

Problem 1

• Explain the difference between biased and unbiased estimators.

• Suppose that \( r \) is a binary random variable. Show that the mean \( E[r] \) is the same as \( \Pr \{ r = 1 \} \).

• Suppose that \( r \) and \( r_i \) (\( i = 1, 2, \ldots \)) are independently identically distributed binary random variables with mean \( \mu = \Pr \{ r = 1 \} \). Let \( X_n = \frac{1}{n} \sum_{i=1}^{n} r_i \). We have the sense that \( X_n \) “approaches \( \mu \).” More formally,
  
  – Show \( E[X_n] = \mu \).
  
  – Compute \( \text{Var}(X_n) \), and show \( \lim_{n \to \infty} \text{Var}(X_n) = 0 \).
  
  – Show that for any \( \epsilon > 0 \),
    \[
    \lim_{n \to \infty} \Pr \{ |X_n - \mu| > \epsilon \} = 0.
    \]

  Hint: Remember Chebyshev’s Inequality from CS22?

• Is \( X_n \) a biased estimator of \( \Pr \{ r = 1 \} \)?

• Is photon mapping’s solution to the rendering equation biased?

Problem 2

Given that a photon is emitted by the photon-mapping algorithm, (i.e., that a photon is emitted from a randomly chosen light source, etc.), derive the probability density that the photon is emitted from point \( x \) on light \( L \) in direction \( \omega \), in terms of the power, area, and normal of \( L \) and of other emitters in the scene. Assume all emitters are uniform lambertian planar sources, and the \( i \)th emitter has area \( A_i \).

Problem 3

What are the probabilities of specular reflection, transmission, diffuse reflection, and absorption of a photon on a surface that is both specular and diffuse? Specular color is \( (S_r, S_g, S_b) \), diffuse color is \( (D_r, D_g, D_b) \), and transmissive color is \( (T_r, T_g, T_b) \). Also \( S + D + T < (1.0, 1.0, 1.0) \).
Problem 4

When a probabilistic photon group ("photon" in photon tracing) is reflected, the energy of the photon is scaled by the probability of reflection. For example, when white light bounces off a blue surface it becomes mostly blue light. Given our Russian roulette setup, derive an expression for the energy of an outgoing photon given the energy of the incoming photon and the reflectivity of the surface. You may assume that there are only three wavelengths.

Problem 5

- Why do we store the *incoming* direction of photons and not the outgoing direction?
- When we compute the radiance estimate at a point during the ray tracing phase of photon mapping, each photon within the disk where we gather acts like a little directional light. Why don’t we divide by the number of photons in the disk?