April 1st

Theorem / There are infinitely many prime numbers.

Proof / Assume not. $\{p_1, \ldots, p_n\}$ ← $n \neq 0$

$N = (p_1 \cdot \ldots \cdot p_n) + 1$ ← 2

$N > p_i$ — N is not divisible by p_i

Contradiction! □.

Human: This is a proof. I am convinced!

SOCIAL OBJECT

- Auto. check a proof.
- Auto. search proof space

Jan - March: Satisfiability

$(P \vee Q)$ ← Find an instance ($\vee$ ← "or")

Now: Validity

$(X \wedge Y) \Rightarrow X$ All instances

(Sat) → Search for CE

(pf) → Search for a formal proof

Natural Deduction

Thm $(X \wedge Y) \Rightarrow X$

$X \wedge Y \vdash X$

premises — $\vdash$ "proves" (syntactically, in our deductive system) — conclusion

$$\frac{A \wedge B}{A} \wedge E_1 \qquad \frac{A \wedge B}{B} \wedge E_2 \qquad \frac{A \quad B}{A \wedge B} \wedge I$$

proof rule: top = premises, bottom = conclusions

$$\frac{X \wedge Y}{X} \wedge E_1$$

□ ← logician-speak for "I'm Done" (empty clause) - ... QED

A proof of $X \wedge Y \vdash X$

Thm / $(X \wedge Y) \Rightarrow (Y \wedge X)$

$$\frac{A \qquad A \Rightarrow B}{B} \Rightarrow_E \text{(MP)} \qquad \frac{\begin{matrix} A \\ \vdots \\ B \end{matrix}}{A \Rightarrow B} \Rightarrow_I$$

Name: "Implication Elimination"

*? $(B \vee \neg A)$

*? $C \wedge, C \Rightarrow A \Rightarrow B$

*? $A \vdash B$

$$\cfrac{\cfrac{\cfrac{X \wedge Y \text{ (assume)}}{Y} \wedge E_2 \qquad \cfrac{X \wedge Y \text{ (assume)}}{X} \wedge E_1}{Y \wedge X} \wedge I}{(X \wedge Y) \Rightarrow (Y \wedge X)} \Rightarrow I$$

$X \wedge Y \;\vdots\; Y \wedge X$

$A \;\vdots\; B$ — $X \wedge Y \;\vdots\; Y \wedge X$

□