Announcements

- Assignment 3 due today!
- Project 1 due today!
- Project 2 out today, due 10/9. 10/16.
Flipping

- Props upside-down.
- Motors spinning the wrong way.
- Motors plugged into the wrong PWM.
Why am I doing this step?

- If you don’t understand why you are doing something in the build, please ask!
Transformations
Rotations

- Diebel, 2006. Representing Attitude: Euler Angles, Unit Quaternions, and Rotation Vectors
Representing Rotations

- Angle (degrees or radians)
- Complex Number
- Rotation Matrix
2.1 Coordinate Transformations

We define the rotation matrix that encodes the attitude of a rigid body to be the matrix that when pre-multiplied by a vector expressed in the world coordinates yields the same vector expressed in the body-fixed coordinates. That is, if \( \mathbf{z} \in \mathbb{R}^3 \) is a vector in the world coordinates and \( \mathbf{z}' \in \mathbb{R}^3 \) is the same vector expressed in the body-fixed coordinates, then the following relations hold:

\[
\mathbf{z}' = R \mathbf{z} \tag{4}
\]
\[
\mathbf{z} = R^T \mathbf{z}' \tag{5}
\]

These expressions apply to vectors, relative quantities lacking a position in space. To transform a point from one coordinate system to the other we must subtract the offset to the origin of the target coordinate system before applying the rotation matrix. Thus, if \( \mathbf{x} \in \mathbb{R}^3 \) is a point in the world coordinates and \( \mathbf{x}' \in \mathbb{R}^3 \) is the same point expressed in the body-fixed coordinates, then we may write

\[
\mathbf{x}' = R (\mathbf{x} - \mathbf{x}_b) = R \mathbf{x} + \mathbf{x}'_w \tag{6}
\]
\[
\mathbf{x} = R^T (\mathbf{x}' - \mathbf{x}'_w) = R^T \mathbf{x}' + \mathbf{x}_b. \tag{7}
\]

Substituting \( \mathbf{x} = 0 \) into Eq. 6 and \( \mathbf{x}' = 0 \) into Eq. 7 yields

\[
\mathbf{x}'_w = -R \mathbf{x}_b \tag{8}
\]
\[
\mathbf{x}_b = -R^T \mathbf{x}'_w. \tag{9}
\]
2.2 Transformation Matrix

It is quite common in the computer graphics community to write Eqs. 6 and 7 as matrix-vector products:

\[
\begin{bmatrix}
    \mathbf{x}' \\
    1
\end{bmatrix} = \begin{bmatrix}
    R & \mathbf{x}'_w \\
    \mathbf{0}^T & 1
\end{bmatrix} \begin{bmatrix}
    \mathbf{x} \\
    1
\end{bmatrix} \tag{10}
\]

\[
= \begin{bmatrix}
    R & -\mathbf{Rx}_b \\
    \mathbf{0}^T & 1
\end{bmatrix} \begin{bmatrix}
    \mathbf{x} \\
    1
\end{bmatrix} \tag{11}
\]

\[
\begin{bmatrix}
    \mathbf{x} \\
    1
\end{bmatrix} = \begin{bmatrix}
    \mathbf{R}^T & \mathbf{x}_b \\
    \mathbf{0}^T & 1
\end{bmatrix} \begin{bmatrix}
    \mathbf{x}' \\
    1
\end{bmatrix} \tag{12}
\]

\[
= \begin{bmatrix}
    \mathbf{R}^T & -\mathbf{R}^T\mathbf{x}'_w \\
    \mathbf{0}^T & 1
\end{bmatrix} \begin{bmatrix}
    \mathbf{x}' \\
    1
\end{bmatrix} \tag{13}
\]

The substantial popularity of this convention is probably due to its adoption by the manufacturers of 3D-accelerated graphics hardware.
Euler Angles $\leq$ Rotation Matrix

$$
\mathbf{u}_{123}(R) = \begin{bmatrix}
\phi_{123}(R) \\
\theta_{123}(R) \\
\psi_{123}(R)
\end{bmatrix} = \begin{bmatrix}
\text{atan2}(r_{23}, r_{33}) \\
-\text{asin}(r_{13}) \\
\text{atan2}(r_{12}, r_{11})
\end{bmatrix}
$$

(72)
Euler Angles => Rotation Matrix

\[ R_{123}(\phi, \theta, \psi) = R_1(\phi)R_2(\theta)R_3(\psi) = \]

\[
\begin{bmatrix}
    c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\
    s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\theta s_\phi \\
    c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\theta c_\phi
\end{bmatrix}
\] (67)
Complex Number

\[ a + bi \]
\[ \sqrt{a^2 + b^2} = 1 \]

\[ \theta = \text{atan} \ 2(b, a) \]
Quaternions

• Complex Numbers on the sphere.

\[ w + xi + yj + zk \]

\[ w \]
\[ x \]
\[ y \]
\[ z \]
Quaternions

\[ q_{123}(\phi, \theta, \psi) = \begin{bmatrix} c_\phi/2c_\theta/2c_\psi/2 + s_\phi/2s_\theta/2s_\psi/2 \\ -c_\phi/2s_\theta/2s_\psi/2 + c_\theta/2c_\psi/2s_\phi/2 \\ c_\phi/2c_\psi/2s_\theta/2 + s_\phi/2c_\theta/2s_\psi/2 \\ c_\phi/2c_\theta/2s_\psi/2 - s_\phi/2c_\psi/2s_\theta/2 \end{bmatrix} \quad (84) \]

\[ q \cdot p = q_m(q, p) \quad (101) \]

\[ = \begin{bmatrix} q_0p_0 - q_{1:3}^Tp_{1:3} \\ q_0p_{1:3} + p_0q_{1:3} - q_{1:3} \times p_{1:3} \end{bmatrix} \quad (102) \]

\[ = \begin{bmatrix} q_0 & -q_{1:3}^T \\ q_{1:3} & q_0I_3 - C(q_{1:3}) \end{bmatrix} \begin{bmatrix} p_0 \\ p_{1:3} \end{bmatrix} \quad (103) \]

\[ = \begin{bmatrix} p_0 & -p_{1:3}^T \\ p_{1:3} & p_0I_3 + C(p_{1:3}) \end{bmatrix} \begin{bmatrix} q_0 \\ q_{1:3} \end{bmatrix}, \quad (104) \]
Unit Quaternion

\[
q_{123}(\phi, \theta, \psi) = \begin{bmatrix}
\frac{c\phi}{2}c\theta/2c\psi/2 + \frac{s\phi}{2}s\theta/2s\psi/2 \\
-\frac{c\phi}{2}s\theta/2s\psi/2 + \frac{c\theta}{2}c\psi/2s\phi/2 \\
\frac{c\phi}{2}c\psi/2s\theta/2 + \frac{c\phi}{2}s\theta/2s\psi/2 \\
\frac{c\phi}{2}s\theta/2s\psi/2 - \frac{s\phi}{2}c\psi/2s\theta/2
\end{bmatrix}.
\] (84)

\[
u_{123}(R_q(q)) = \begin{bmatrix}
\text{atan2}(2q_2q_3 + 2q_0q_1, \sqrt{q_3^2 - q_2^2 - q_1^2 + q_0^2}) \\
-\text{asin}(2q_1q_3 - 2q_0q_2) \\
\text{atan2}(2q_1q_2 + 2q_0q_3, \sqrt{q_1^2 + q_0^2 - q_3^2 - q_2^2})
\end{bmatrix}.
\] (290)
Uncertainty
Uncertainty

- Deriving the rules of probability.
- Conditional independence in discrete states.
- Bayes Filters.
  - Kalman Filters
  - Particle Filters
  - SLAM
- Measurement models.
- Motion models.
Probability Theory: The Logic of Science

E.T. Jaynes
Deductive Reasoning

• If $A$ is true then $B$ is true.
  $A$ is true.
  Therefore, $B$ is true.

• If $A$ is true then $B$ is true.
  $B$ is false.
  Therefore, $A$ is false.
Plausible Reasoning

- If $A$ is true, then $B$ is true.
  
  $B$ is true.
  
  Therefore, $A$ becomes more plausible.

- If $A$ is true, then $B$ is true.
  
  $A$ is false.
  
  Therefore, $B$ becomes less plausible.

- If $A$ is true, then $B$ becomes more plausible.
  
  $B$ is true.
  
  Therefore, $A$ becomes more plausible.
Desiderata

- Degrees of plausibility are represented by real numbers.
- Qualitative correspondence with common sense.
- Consistency.
  - If a conclusion can be reasoned out in more than one way, then every possible way must lead to the same result.
  - The robot always takes into account all of the evidence it has that is relevant to a question. It does not arbitrarily ignore some of the information, basing its conclusions on only what remains. In other words, the robot is completely non-ideological.
  - The robot always represents equal states of knowledge by equal plausibility assignments. That is, if in two problems the robot's state of knowledge is the same (except perhaps for the labeling of the propositions), then it must assign the same plausibilities in both.
Desiderata

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  - The robot always takes into account all of the evidence it has that is relevant to a question. It does not arbitrarily ignore some of the information, basing its conclusions on only what remains. In other words, the robot is completely non-ideological.
  - The robot always represents equal states of knowledge by equal plausibility assignments. That is, if in two problems the robot's state of knowledge is the same (except perhaps for the labeling of the propositions), then it must assign the same plausibilities in both.

\[
p(AB|C) = p(A|BC) \cdot p(B|C)
\]

\[
p(A|B) + p(\neg A|B) = 1
\]
Bayes’ Rule

\[ p(A|B) = \frac{p(A, B)}{p(B)} \]
Where are you?
Localization (Discrete)

Observations:
I see a chair.

Kitchen?
Living room?
Bedroom?
Localization (Discrete)

Observations:
I see a chair.
I see a bed.

Kitchen?
Living room?
Bedroom?
Localization (Discrete)

Observations:
I see a chair.
I see a microwave.
Formalizing

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a chair.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a bed.} \]
\[ E = \text{I see a sofa.} \]

How can we compute this?

\[ P(A = A_1 | BC) \]
Compute Directly?

\[
\begin{align*}
P(A & = A_1 | B) \\
P(A & = A_1 | BC) \\
P(A & = A_1 | BD) \\
P(A & = A_1 | BE) \\
P(A & = A_1 | C) \\
P(A & = A_1 | CD) \\
P(A & = A_1 | CE) \\
P(A & = A_1 | D) \\
P(A & = A_1 | DE) \\
P(A & = A_1 | E) \\
P(A & = A_1 | BCD) \\
P(A & = A_1 | CDE) \\
P(A & = A_1 | BCE) \\
\end{align*}
\]

\[A_1 = \text{I'm in the kitchen.}\]
\[A_2 = \text{I'm in the bedroom.}\]
\[A_3 = \text{I'm in the living room.}\]
\[B = \text{I see a chair.}\]
\[C = \text{I see a microwave.}\]
\[D = \text{I see a bed.}\]
\[E = \text{I see a sofa.}\]
Joint Distribution

\[
P(A_1, A_2, A_3, B, C, D, E) \\
P(A = A_i, B, C, D, E) \\
P(A, B, C, D, E)
\]

- Mutually exclusive.
- Exhaustive.

\[
A_1 = \text{I'm in the kitchen.} \\
A_2 = \text{I'm in the bedroom.} \\
A_3 = \text{I'm in the living room.} \\
B = \text{I see a bed.} \\
C = \text{I see a microwave.} \\
D = \text{I see a chair.} \\
E = \text{I see a sofa.}
\]
Joint Distribution

\[ P(A, B) \]

<table>
<thead>
<tr>
<th>A=kitchen</th>
<th>A=living room</th>
<th>A=bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>I see a bed</td>
<td></td>
<td>I'm in the bedroom.</td>
</tr>
<tr>
<td>I don't see a bed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\sum_A \sum_B P(A, B) = A_1 = \text{I'm in the kitchen.} \\
A_2 = \text{I'm in the bedroom.} \\
A_3 = \text{I'm in the living room.} \\
B = \text{I see a bed.} \\
C = \text{I see a microwave.} \\
D = \text{I see a chair.} \\
E = \text{I see a sofa.} \\
\]
Joint Distribution

\[ P(A, B) \]

\begin{align*}
\begin{array}{ccc}
& A=\text{kitchen} & A=\text{living room} & A=\text{bedroom} \\
\text{I see a bed} & 0.1 & 0.1 & 0.3 \\
\text{I don't see a bed} & 0.2 & 0.2 & 0.1 \\
\end{array}
\end{align*}

\[ \sum_A \sum_B P(A, B) = 1 \]

\( A_1 = \text{I'm in the kitchen.} \)
\( A_2 = \text{I'm in the bedroom.} \)
\( A_3 = \text{I'm in the living room.} \)
\( B = \text{I see a bed.} \)
\( C = \text{I see a microwave.} \)
\( D = \text{I see a chair.} \)
\( E = \text{I see a sofa.} \)
Conditional Distribution

\[ P(A, B) = P(A|B)P(B) \]

\[ P(A|B) = \]

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
Marginalizing

\[ P(B) = \sum A P(A, B) + p(\neg X|Y) = 1 \]

Law of Total Probability

\[ p(X|Y) + p(\neg X|Y) = 1 \]

\( A_1 = \) I'm in the kitchen.
\( A_2 = \) I'm in the bedroom.
\( A_3 = \) I'm in the living room.
\( B = \) I see a bed.
\( C = \) I see a microwave.
\( D = \) I see a chair.
\( E = \) I see a sofa.
Marginal Distributions

\[ P(B) = \sum_A P(A, B) \]

I see a bed
I don't see a bed

\[ P(A, B) \]

<table>
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<th>A=kitchen</th>
<th>A=living room</th>
<th>A=bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>I see a bed</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>I don't see a bed</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\( A_1 = \) I'm in the kitchen.
\( A_2 = \) I'm in the bedroom.
\( A_3 = \) I'm in the living room.
\( B = \) I see a bed.
\( C = \) I see a microwave.
\( D = \) I see a chair.
\( E = \) I see a sofa.
Marginal Distribution

\[ P(A) \]

<table>
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<th>A=living room</th>
<th>A=bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>I see a bed</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>I don't see a bed</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ P(A, B) \]

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
Conditional Distribution

\[ P(A|B) = \frac{P(A, B)}{\sum_{A'} P(A', B)} \]

<table>
<thead>
<tr>
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<th>A=kitchen</th>
<th>A=living room</th>
<th>A=bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>I see a bed</td>
<td>0.1</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>I don't see a bed</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ P(A, B) \]

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
Conditional Distribution

\[ P(B|A) \]

<table>
<thead>
<tr>
<th></th>
<th>A=kitchen</th>
<th>A=living room</th>
<th>A=bedroom</th>
</tr>
</thead>
<tbody>
<tr>
<td>I see a bed</td>
<td>0.33333</td>
<td>0.33333</td>
<td>0.75</td>
</tr>
<tr>
<td>I don't see a bed</td>
<td>0.66667</td>
<td>0.66667</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Formalizing

$A_1 = \text{I'm in the kitchen.}$
$A_2 = \text{I'm in the bedroom.}$
$A_3 = \text{I'm in the living room.}$
$B = \text{I see a bed.}$
$C = \text{I see a microwave.}$
$D = \text{I see a chair.}$
$E = \text{I see a sofa.}$

$P(A = A_1 | BC)$

How can we compute this?
How Big is the Table?

\[ P(A, B, C, D, E) \]

(Actually 47 because things have to sum to 1.)

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
How Big is the Table?

\[ P(A, B, C) \]

11 entries because things sum to 1

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
Independence

Two random variables are independent iff \( P(A|B) = P(A) \)

\[
P(A, B) =
\]

\[
P(A, B) =
\]
Conditional Independence

X is conditionally independent of Y given Z if

\[ P(X \mid Y, Z) = P(X \mid Z) \]

\[ P(A, B, C) = \]

\[ P(A, B, C) = \]

\[ A = \text{I'm in the kitchen}. \]
\[ B = \text{I see a bed}. \]
\[ C = \text{I see a microwave}. \]
How many entries?

\[ P(A, B, C) = P(A) P(B|A) P(C|A) \]

\[ A_1 = \text{I'm in the kitchen.} \]
\[ A_2 = \text{I'm in the bedroom.} \]
\[ A_3 = \text{I'm in the living room.} \]
\[ B = \text{I see a bed.} \]
\[ C = \text{I see a microwave.} \]
\[ D = \text{I see a chair.} \]
\[ E = \text{I see a sofa.} \]
Graphical Notation

\[ A = I'm \text{ in the kitchen.} \]
\[ B = I \text{ see a bed.} \]
\[ C = I \text{ see a microwave.} \]

\[ P(A, B, C) = P(A)P(B|A)P(C|A) \]
Graphical Notation

$A = \text{I have a cavity.}$
$B = \text{I have a toothache.}$
$C = \text{A steel probe catches in my tooth.}$

$$P(A, B, C) = P(A) P(B|A) P(C|A)$$
Formalizing

\[ A_1 = \text{I'm in the kitchen}. \]
\[ A_2 = \text{I'm in the bedroom}. \]
\[ A_3 = \text{I'm in the living room}. \]
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\[ C = \text{I see a microwave}. \]
\[ D = \text{I see a chair}. \]
\[ E = \text{I see a sofa}. \]

\[ P(A = A_1 | BC) \]

How can we compute this?
\[ P(A,B,C,D,E) = P(A)P(B|A)P(C|A)P(D|A)P(E|A) \]
### Tables

|                  | $P(A)$      | $P(B|A)$ | $P(C|A)$ |
|------------------|-------------|----------|----------|
| $A=$kitchen      | 0.33333     | 0.8      | 0.1      |
| $A=$living room  | 0.33333     | 0.1      | 0.1      |
| $A=$bedroom      | 0.33333     | 0.9      | 0.2      |

**I see a bed**
- $P(A) = 0.33333$
- $P(B|A) = 0.8$
- $P(C|A) = 0.1$

**I don't see a bed**
- $P(A) = 0.33333$
- $P(B|A) = 0.1$
- $P(C|A) = 0.9$

**I see a microwave**
- $P(A) = 0.33333$
- $P(B|A) = 0.9$
- $P(C|A) = 0.1$

**I don't see a microwave**
- $P(A) = 0.33333$
- $P(B|A) = 0.1$
- $P(C|A) = 0.9$

$$P(A, B, C, D, E) = P(A)P(B|A)P(C|A)P(D|A)P(E|A)$$
How can we compute?

\[ P(A|B) = \]

\[ P(A|B) = \frac{\sum C P(A, B, C)}{\sum A, C P(A, B, C)} \]

\[ P(A|B) = \frac{\sum C P(A) P(B|A) P(C|A)}{\sum A, C P(A) P(B|A) P(C|A)} \]
How can we compute?

\[ P(A|BC) = \]

\[ P(A|BC) = \frac{P(A, B, C)}{\sum_A P(A, B, C)} \]

\[ P(A|BC) = \frac{P(A) P(B|A) P(C|A)}{\sum_A P(A) P(B|A) P(C|A)} \]
Bayes' Rule

\[ P(A, B) = P(B|A) P(A) \]
\[ P(A, B) = P(A|B) P(B) \]
\[ P(A|B) = \frac{P(B|A) P(A)}{P(B)} \]
Summary

- Independence assumptions make computation easier.
- The game: pick the independence assumptions that make computation tractable without sacrificing accuracy.
Localization

$A_1 = \text{I'm in the kitchen.}$
$A_2 = \text{I'm in the bedroom.}$
$A_3 = \text{I'm in the living room.}$
$B = \text{I see a chair.}$
$C = \text{I see a microwave.}$
$D = \text{I see a bed.}$
$E = \text{I see a sofa.}$

$P(A = A_1|BC)$

How can we compute this?
Localization

$x_t$: State at time $t$
$z_t$: Measurement at time $t$
$u_t$: Control input at time $t$

Where am I?

We want to know: $p(x_t | z_{1:t}, u_{1:t})$

$p(x_t | x_{0:t-1}, z_{0:t-1}, u_{1:t}) = p(x_t | x_{t-1}, u_t)$ State transition probability

$p(z_t | x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t | x_t)$ Measurement probability
**Bayesian Filtering**

\[
p(x_t|\mathbf{x}_{0:t-1}, \mathbf{z}_{0:t-1}, \mathbf{u}_{1:t}) = p(x_t|x_{t-1}, u_t) \quad \text{State transition probability}
\]

\[
p(z_t|\mathbf{x}_{0:t}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) = p(z_t|x_t) \quad \text{Measurement probability}
\]
Bayesian Filtering

We want to know: \( \text{bel}(x_t) = p(x_t | z_{1:t}, u_{1:t}) \)
1: \textbf{Algorithm Bayes\_filter}(bel(x_{t-1}), u_t, z_t):
2: \hspace{1em} for all $x_t$ do
3: \hspace{2em} $\overline{bel}(x_t) = \int p(x_t | u_t, x_{t-1}) \, bel(x_{t-1}) \, dx$
4: \hspace{2em} $bel(x_t) = \eta \, p(z_t | x_t) \, \overline{bel}(x_t)$
5: \hspace{1em} endfor
6: \hspace{1em} return $bel(x_t)$
Prediction

\[ p(x_t|z_{1:t-1}, u_{1:t}) = \int p(x_t, x_{t-1}|z_{1:t-1}, u_{1:t}) \, dx_{t-1} \]  

Sum rule

\[ p(x_t|z_{1:t-1}, u_{1:t}) = \int p(x_t|x_{t-1}, z_{1:t-1}, u_{1:t}) \, p(x_{t-1}|z_{1:t-1}, u_{1:t}) \, dx_{t-1} \]  

Product rule

\[ p(x_t|z_{1:t-1}, u_{1:t}) = \int p(x_t|u_t, x_{t-1}) \, p(x_{t-1}|z_{1:t-2}, u_{1:t-1}) \, dx_{t-1} \]  

Conditional Independence

\[ \bar{\text{bel}}(x_t) = \int p(x_t|u_t, x_{t-1}) \, \text{bel}(x_{t-1}) \, dx_{t-1} \]
1: **Algorithm Bayes_filter**(\(bel(x_{t-1}), u_t, z_t\)):  
2: for all \(x_t\) do  
3: \[bel(x_t) = \int p(x_t | u_t, x_{t-1}) \; bel(x_{t-1}) \; dx\]  
4: \[bel(x_t) = \eta \; p(z_t | x_t) \; \overline{bel(x_t)}\]  
5: endfor  
6: return \(bel(x_t)\)
Measurement Update

$$p(x_t|z_{1:t}, u_{1:t}) = \frac{p(z_t|x_t, z_{1:t-1}, u_{1:t}) p(x_t|z_{1:t-1}, u_{1:t})}{p(z_t|z_{1:t-1}, u_{1:t})}$$

Bayes' rule

Conditional independence

$$p(x_t|z_{1:t}, u_{1:t}) = \frac{p(z_t|x_t) p(x_t|z_{1:t-1}, u_{1:t})}{p(z_t|z_{1:t-1}, u_{1:t})}$$

$$p(x_t|z_{1:t}, u_{1:t}) = \eta p(z_t|x_t) p(x_t|z_{1:t-1}, u_{1:t})$$

$$bel(x_t) = \eta p(z_t|x_t) \tilde{bel}(x_t)$$
Localization

Where am I?

\[ x_t : \text{State at time } t \]
\[ z_t : \text{Measurement at time } t \]
\[ u_t : \text{Control input at time } t \]

We want to know: \[ p(x_t|z_{1:t}, u_{1:t}) \]

State transition probability
\[ p(x_t|x_{0:t-1}, z_{0:t-1}, u_{1:t}) = p(x_t|x_{t-1}, u_t) \]

Measurement probability
\[ p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t|x_t) \]
Localization

$x_t$: State at time $t$
$z_t$: Measurement at time $t$
$u_t$: Control input at time $t$

We want to know: $p(x_t|z_{1:t}, u_{1:t})$

$p(x_t|x_{0:t-1}, z_{0:t-1}, u_{1:t}) = p(x_t|x_{t-1}, u_t)$ State transition probability

$p(z_t|x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t|x_t)$ Measurement probability
Localization and Mapping

$x_k$: Position and orientation of vehicle at step $k$

$u_k$: Control vector at step $k$

$z_{ki}$: Observed distance and heading to landmark $i$ at step $k$

$m_i$: True location of landmark $i$

We want to know: $p(x_k, m_1...m_n | Z_{0:k}, U_{0:k}, x_0)$
Uncertainty

- Deriving the rules of probability.
- Conditional independence in discrete states.
- Bayes Filters.
  - Kalman Filters
  - Particle Filters
  - SLAM