**k-Vickrey Example**

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Imagine three bidders, $b_1, b_2$ and $b_3$, and 2 items. The auctioneer believes their values to be uniformly distributed with different bounds: $b_i$ has a value uniformly distributed between 0 and $i$, so $f_i(v) = \frac{1}{i}, 0 \leq v \leq i$, and therefore $F_i(v) = \frac{v}{i}, 0 \leq v \leq i$.

Let $v_i$ represent the actual value of bidder $i$. Suppose $v_1 = \frac{3}{4}, v_2 = \frac{3}{2}, v_3 = 1$. Assume bidders bid truthfully. What happens in the auction? Well we have to do the following:

1. calculate the virtual value function for each bidder
2. use the function to find their virtual values
3. sort the virtual values
4. throw out the bidders with negative virtual values
5. find winners
6. calculate critical bid for each winner
7. make them pay critical bid

Steps 1-5 are described in the following table:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$v_i$</th>
<th>$F_i(v)$</th>
<th>$\varphi_i(v)$</th>
<th>$\varphi_i(v_i)$</th>
<th>virtual value rank</th>
<th>$\varphi_i(v_i) \geq 0?$</th>
<th>winner?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{4}$</td>
<td>$v$</td>
<td>$2 * v - 1$</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{v}{2}$</td>
<td>$2 * v - 2$</td>
<td>1</td>
<td>1</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>$1$</td>
<td>$\frac{v}{3}$</td>
<td>$2 * v - 3$</td>
<td>$-1$</td>
<td>3</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

So here we see that bidders 1 and 2 get the item, because they have the two highest virtual values, and neither of their virtual values are negative. But what do they pay? Well they pay their critical bid, which is the smallest bid they would have had to make to get the item (this is Myerson’s payment formula). Well for bidders 1 and 2 to get the item, they certainly have a higher virtual value than bidder 3. But that is not good enough, because bidder 3 has a negative virtual value, specifically $-1$. So if bidder 1 had a virtual value of $-\frac{1}{2}$, they would have the second highest virtual value, but they would not get one of the two items because they but they have a negative virtual value! So to get allocated, bidders 1 and 2 both need to have a virtual value of at least 0. So their critical bid is the bid which would give them a virtual value of 0. The remaining steps are given in the following table.
So bidders 1 and 2 get the item. Bidder 1 pays .5 and bidder 2 pays 1.