We discuss some drawbacks of the Vickrey auction and the VCG mechanism. We then move on to discuss ascending auctions. We note that even ascending auctions must charge VCG payments to achieve desired incentive guarantees. We then describe a few EPIC ascending auctions, as well as a strategy for proving an ascending auction is EPIC.

1 Ascending Auctions

“The Lovely by Lonely Vickrey Auction” is the title of a paper by two prominent economists, Ausubel and Milgrom, ¹ who note that the Vickrey auction is indeed lovely in theory but rarely used in practice.² On the contrary, most auction houses sell their wares via an open-outcry, ascending (a.k.a. English) auction. Why is this?

One likely reason is the fact that bidders rarely have a precise number in mind that articulates what a good might be worth to them. Indeed, for companies bidding on contracts of some sort, it may be computationally intensive to compute such a number. On the other hand, even without knowledge of a precise number, it may be still be possible to answer (so-called demand) queries of the form, “Are you willing to pay more than x for a good?”. Hence, such auctions may be less challenging to bidders, and remember, attracting bidders is an essential step in running a profitable auction.

Second, and arguably even more powerful, bidders tend to engage in bidding wars during English auctions. Even if bidders knew their precise value for the good at hand, they still might bid beyond that value. This behavior is rooted in our psychology; in particular, “losses loom larger than gains”.³ Applied to auctions, this maxim suggests that someone can become attached to a good while they are winning that good, and might therefore be willing to bid higher than their value to hold on to their tentative winnings. Furthermore, some might associate shame with losing, and pride with winning, especially when auction results are made public.

Remark 1.1. It has been said that “the only thing worse than losing an auction is winning.” Regardless of whether they know their own precise value for a good, winning bidders often regret having won an auction, because upon winning, it is revealed to them that their bid was greater than anyone else’s value. If indeed their own value is not definitively greater than their bid, then they may experience buyer’s remorse—a feeling of post-purchase regret, stemming from the fact

² A notable exception were stamp auctions during the 19th century, in which bids were sent via post to auctioneers in sealed envelopes.
that other alternatives are no longer available (for example, because of a reduction in purchasing power).

These phenomena help explain why the English auction is more commonly used in practice than the Vickrey auction.

Last time, we defined the VCG mechanism, which generalizes the Vickrey auction to the multi-parameter setting. This mechanism suffers from several anomalies, which you explored in your homework. In addition, when valuations have exponential complexity, the communication complexity of VCG is likewise exponential, and the computation of a VCG outcome is potentially exponential as well (solving for a welfare-maximizing allocation is NP-hard).

Since the Vickrey auction, in the single-parameter setting, is rarely used in practice, and since VCG, its generalization to the multi-parameter setting, is less than ideal, an alternative model of auctions is needed. In search of such an alternative, we now turn our attention to indirect mechanisms, specifically ascending auctions, in which prices are adjusted over time. For our purposes, an ascending auction is an iterative algorithm that abides by the following rules:

- Prices are per good (as opposed to per bundle).
- Prices are initialized at zero, and can only increase over time.
- Queries take the form of **demand queries**, meaning bidders are asked what set of goods they want at the current prices.
- The final allocation and payments are a (possibly randomized) function of the auction’s history (i.e., the sequence of prices, any tentative allocations, and responses to demand queries), only.

This ascending auction design—specifically, the demand queries—eliminates the communication complexity of VCG (although it may be NP-hard for a bidder to solve a demand query). Other advantages of ascending auctions include: greater transparency; less information about bidders’ valuations is revealed to the auctioneer; price discovery, which can help bidders hone in on their preferred bundles; potentially more revenue, because bidding wars can arise.

2 **Ascending Auctions and VCG**

Recall that the VCG payment made by bidder $i$ can be interpreted as the externality bidder $i$ imposes on the other auction participants. These payments ensure that the mechanism is DSIC. Furthermore, by the “VCG uniqueness” theorem, the only way to design a DSIC auction is to charge each bidder their externality—no more; no less.
More specifically, among all direct mechanisms, VCG is the unique DSIC, welfare-maximizing (WM) mechanism, assuming a technical condition. In other words, no direct mechanism can be DSIC and WM that does not yield a VCG outcome.

The first question we ask, then, as we begin our exploration of incentives in ascending auctions, is: Is this result also true of ascending auctions? That is, is it also the case that no indirect mechanism can be DSIC and WM that does not yield a VCG outcome?

Before we can answer this question, we must define what it means for an ascending auction to be DSIC. And before we can do that, we must define the analog of truthful bidding for ascending auctions.

**Definition 2.1.** A bidding strategy is called sincere if the bidder responds to all queries immediately and truthfully, meaning in a way that reflects their values.

For example, a sincere bidder would answer “yes” straightaway to the query “Would you like the good for $10?” iff their value for the good is at least $10.

**Definition 2.2.** An indirect mechanism is DSIC if sincere bidding is a dominant strategy. Likewise, an indirect mechanism is EPIC or BIC, if sincere bidding is an EPNE or BNE, respectively.

**Theorem 2.3.** If an indirect mechanism is DSIC and welfare-maximizing, then the payments must be VCG payments, i.e., the outcome is a VCG outcome (i.e., a welfare-maximizing allocation and VCG payments).

**Proof.** Apply the revelation principle. The result is a DSIC outcome-preserving (i.e., welfare-maximizing) direct mechanism. But the unique direct DSIC, welfare-maximizing mechanism is VCG. Therefore, the outcome of the indirect mechanism is a VCG outcome.

This theorem yields a strategy for designing ascending auctions that yield a VCG outcome: Simply check that the auction is DSIC and welfare-maximizing, and then boom!—VCG payments come for free. Next, we investigate how this strategy applies in a few examples.

Recall from Homework 4 that the English auction, under the usual activity rules—bidders can come and go as they please—is not DSIC (even up to $\epsilon$, an additive constant). Nonetheless, we can modify the rules so that it is DSIC (up to $\epsilon$). The necessary modification is simply that bid re-entry is forbidden; that is, if a bidder ever skips a round of the auction, they cannot bid again. With this modification, the English auction—in fact, the $k$-good English auction, assuming unit demand—is DSIC (up to $\epsilon$). See Appendix A for details.

Now assume the $k$ goods are distinct, and further, assume additive...
valuations. That is, bidder \( i \)'s value for a bundle of goods \( X \subseteq G \) is:

\[
v_i(X) = \sum_{j \in X} v_i(j)
\]

(1)

Since each bidder’s value for each of the goods is independent of its values for the others, a viable solution might be to run \( k \) parallel (modified) English auctions. While this design is indeed welfare-maximizing, it is not DSIC (even up to \( \epsilon \)), as the next example shows:

**Example 2.4.** Suppose goods \( a \) and \( b \) are being auctioned off in two parallel English auctions, and assume two bidders, \( i \) and \( j \). Let \( v_i(a) = v_i(b) = 3 \), and \( v_j(a) = v_j(b) = 1 \). Suppose bidder \( j \) declares that unless bidder \( i \) forfeits \( a \) on the very first round, they will bid on both goods forever, but if \( i \) does forfeit \( a \), then \( j \) will forfeit \( b \) (which wasn’t rightly theirs to win, anyway). Given this opposing strategy, it is in bidder \( i \)'s interest to (insincerely) make no attempt to procure \( a \) so that they will at least win \( b \), obtaining a utility of 2, rather than 0.

While we are not able to guarantee the DSIC property in parallel English auctions assuming additive valuations, we are able to guarantee the weaker (albeit still rather strong) property of ex-post incentive compatibility: i.e., if all other bidders are sincere, then it is in \( i \)'s best interest to bid sincerely as well. Additive valuations decouple the parallel auctions, as does sincere bidding (bids in one auction are independent of those in any other), so that we can analyze the parallel auctions separately. The following proposition thus follows from our analysis of the \( k \)-good auction, i.e., Propositions A.1 and A.2.

**Proposition 2.5.** Assuming additive valuations over \( k \) distinct goods, running \( k \) parallel English auctions is EPIC and welfare-maximizing.

This proposition holds not only for the modified form of the English auction introduced today, but in addition, for the usual form of the English auction, where bidders can come and go as they please. This latter claim follows from the analysis of the English auction on Homework 4, where it was established that this auction is EPIC.

Now, back to our strategy for designing ascending auctions that yield a VCG outcome. Parallel English auctions, assuming additive valuations, are not DSIC, although they are welfare-maximizing, so does this mean their outcome is not necessarily VCG? It turns out that it does not, because of a key lemma we proved back in Lecture 3, namely that EPIC and DSIC coincide in direct mechanisms.

**Theorem 2.6.** If an indirect mechanism is EPIC and welfare-maximizing, then the payments must be VCG payments: i.e., the outcome is a VCG outcome (i.e., a welfare-maximizing allocation and VCG payments).
Proof. Apply the revelation principle. The result is an EPIC outcome-preserving (i.e., welfare-maximizing) direct mechanism. But EPIC and DSIC coincide in direct mechanisms, and the unique direct DSIC, welfare-maximizing mechanism is VCG. Therefore, the outcome of the indirect mechanism is a VCG outcome.

In sum, a strategy for proving an ascending auction yields a VCG outcome is to show that it is EPIC and welfare-maximizing and then boom!—VCG payments come for free.

3 Ascending Auctions and Incentives

While it is nice to know that certain ascending auctions can recover the VCG outcome, what we would really like to know is how to design ascending auctions with incentive guarantees. Hence, we are driven to investigate the converse of the conclusion to the previous discussion: if an ascending auction yields a VCG outcome, is it EPIC?

If only life were so simple . . . the answer to this question is no. Consider the usual English auction with the following funky modification:

If when the price is \( q \), a bidder does not bid, but then when the price is \( q + \epsilon \), the bidder does bid, the auction ends, and that bidder wins the good at the price \( q \).

In this funky English auction, sincere bidding yields the VCG outcome. However, if all other bidders are sincere, it is better for a bidder to sit out the first round, and then win the good in the second round at price 0. Therefore, for an auction to be EPIC, it is not enough for it to yield the VCG outcome.

We will resolve this issue by supplying an additional condition, beyond a mere VCG outcome, that guarantees EPIC. Before doing so, we restrict the space of bidding strategies to derive an equivalence between DSIC and EPIC in ascending auctions. (Recall that these properties are equivalent in direct, but not indirect, mechanisms.)

**Definition 3.1.** A strategy \( c_i \) for bidder \( i \) is called **consistent** iff there exists a valuation for which \( c_i \) is sincere.

We denote the space of bidder \( i \)’s consistent strategies by \( C_i \).

**Definition 3.2.** A strategy \( s_i \) for bidder \( i \in N \) is called **dominant up to consistency** if it is (weakly) optimal relative to all other consistent strategies, regardless of all the other bidders’ strategies and types, but again assuming only consistent strategies, even on the part of other bidders: i.e., for all bidders \( i \in N \),

\[
u_i(s_i(v_i), c_{-i}(v_{-i})) \geq u_i(c_i(v_i), c_{-i}(v_{-i})), \quad \forall c_i \in S_i, \forall c_{-i} \in C_{-i}, \forall v \in T.
\]
In words, a strategy is DSIC up to consistency if bidding sincerely is an ex-post best response for each bidder relative to other consistent strategies, regardless of how the other agents bid, assuming they, too, bid consistently.

**Definition 3.3.** A strategy profile comprises a dominant strategy equilibrium up to consistency if all bidders’ strategies are dominant up to consistency. An auction is called dominant-strategy incentive compatible up to consistency if sincere bidding is a dominant strategy up to consistency.

**Proposition 3.4.** If an indirect mechanism is EPIC up to consistency, then it is also DSIC up to consistency.

**Proof.** Since \( M \) is EPIC up to consistency, for all bidders \( i \in N \) and for all (true) value profiles \( v \in T \), the sincere bidding profile \( s \in S \) satisfies

\[
u_i(s_i(v), s - i(v - i)) \geq u_i(c_i(v), s - i(v - i)), \quad \forall c_i \in S_i.
\]

Our goal is to show that \( M \) is also DSIC up to consistency: i.e., for all bidders \( i \in N \) and for all (true) values \( v_i \in T_i \)

\[
u_i(s_i(v_i), c - i(v - i)) \geq u_i(c_i(v_i), c - i(v - i)), \quad \forall c_i \in S_i, \forall c - i \in C - i, \forall v - i \in T - i.
\]

Fix a bidder \( i \) and their true value \( v_i \). For two arbitrary value profiles \( v' - i, v'' - i \in T \), since \( M \) is EPIC up to consistency,

\[
u_i(s_i(v_i), s - i(v' - i)) \geq u_i(c_i(v_i), s - i(v' - i)), \quad \forall c_i \in S_i.
\]

\[
u_i(s_i(v_i), s - i(v'' - i)) \geq u_i(c_i(v_i), s - i(v'' - i)), \quad \forall c_i \in S_i.
\]

Hence, sincere bidding is a best response for bidder \( i \), relative to other consistent strategies, assuming others are also bidding sincerely, regardless their value profiles. But if all other bidders are bidding sincerely relative to some value profile or another, then they are bidding consistently! In other words, sincere bidding is a dominant-strategy up to consistency: i.e.,

\[
u_i(s_i(v_i), c - i(v - i)) \geq u_i(c_i(v_i), c - i(v - i)), \quad \forall c_i \in S_i, \forall c - i \in C - i, \forall v - i \in T - i.
\]

Since bidder \( i \) was arbitrary, sincere bidding is a dominant-strategy equilibrium up to consistency (i.e., it is a best response for all bidders up to consistency). 

Therefore, just as the notions of DSIC and EPIC coincide in direct mechanisms, DSIC and EPIC, up to consistency, coincide in indirect mechanisms. We use both these facts to prove the next lemma:
Lemma 3.5. If sincere bidding in an indirect mechanism yields the VCG outcome, then it is DSIC/EPIC up to consistency.

Proof. Apply the revelation principle. The result is a direct mechanism in which truthful bidding yields the VCG outcome. Any such mechanism is DSIC and EPIC, meaning truthful bidding is an equilibrium regardless of types. So likewise, in the indirect mechanism, sincere bidding must have been an equilibrium regardless of types: i.e., sincere bidding must have been a best-reponse relative to all strategies that depend only on agents’ types: i.e., relative to all consistent strategies. In other words, the indirect mechanism must have been DSIC and EPIC, up to consistency.

The following theorem follows immediately from this lemma:

Theorem 3.6. If sincere bidding in an indirect mechanism yields the VCG outcome, then it is EPIC, as long as no inconsistent strategy yields greater utility than consistent bidding for any one bidder, assuming all the others bid sincerely.

Remark 3.7. The following is also true, however it does not have much bite, since relatively few indirect mechanisms are in fact DSIC: If sincere bidding in an indirect mechanism yields the VCG outcome, then it is DSIC, as long as no inconsistent strategy yields greater utility than consistent bidding for any bidder, assuming arbitrary bidding on the part of others.

Theorem 3.6 yields a recipe for designing EPIC indirect auctions:

1. Show that sincere bidding yields the VCG outcome: i.e., that it is welfare-maximizing and yields VCG payments.
2. Argue that no inconsistent strategy yields greater utility than consistent bidding for any bidder.

We apply this recipe in the next lecture.

A k-Good English Auction

The k-good English auction can be defined as follows:

• Initialize the set of bidders $S_0 = N$ and the price $p_0 = 0$.
• At each round $i = 1, 2, \ldots$,
  - Let price $p_i = i e$.
  - Let $S_i$ be the bidders in $S_{i-1}$ who remains interested at price $p_i$. N.B. No bidder who expressed disinterest earlier can re-express their interest at some later round.

We cannot say anything about how effective a strategy may have been relative to inconsistent strategies.
• Increment $i$ until $|S_i| \leq k$. Call the final round $m$.
  
  – Give $|S_m|$ of the $k$ goods to the bidders in $S_m$ at price $(m-1)\epsilon$.
  
  – Give the remaining $k - |S_m|$ of the goods (if any) to random bidders in $S_{m-1} \setminus S_m$ at price $(m-1)\epsilon$.

This $k$-good English auction is DSIC up to $\epsilon$, for an appropriate choice of $\epsilon$—assuming unit demand valuations.

**Proposition A.1.** In the $k$-good English auction assuming unit demand, sincere bidding is a dominant strategy, up to $\epsilon$.\(^9\)

**Proof.** Fix bidder $i$ and all other valuations $v_{-i}$. Let $j, j'$ be such that $je \leq v_i < j'e$. Then sincere bidding would consist of a reply of “yes” during round $j$ (i.e., “yes, I want the good for $je$”) and “no” during round $j'e$. (i.e., “no, I don’t want the good for $j'e$”). We must show that replying “yes” during round $j$ is at least as good (up to $\epsilon$) as replying “no,” and vice versa, during round $j'$. Suppose it is round $j$. If bidder $i$ replies “yes,” they may win the good at price $je$ or $(j-1)e$, or they may not win the good this round. The first two options yield non-negative utility: $v_i - je \geq 0$ and $v_i - (j-1)e > 0$, respectively. If bidder $i$ replies “no,” they obtain 0 utility, so it is no worse to reply sincerely (“yes”).

Suppose it is round $j'$. If bidder $i$ replies “no,” they obtain 0 utility. If they reply “yes,” they may win the good at price $j'e$ or $(j'-1)e$, or they may not win the good this round. In the former case, their utility is $v_i - j'e < 0$, but it could be that $v_i - (j'-1)e > 0$. However, $v_i - (j'-1)e = (v_i - j'e) + e < e$ (since $v_i - j'e < 0$). Therefore, by bidding insincerely, bidder $i$ can obtain utility of at most $\epsilon$.

This argument was independent of bidders’ strategies, other than bidder $i$’s; hence, sincere bidding is a dominant strategy, up to $\epsilon$. \(\square\)

**Proposition A.2.** Assuming sincere bidding in a $k$-good English auction, the total welfare is within $k\epsilon$ of the optimal.

**Proof.** Let $A$ denote the set of bidders with the highest $k$ values,\(^10\) and let $B$ be the set of winners. Note that regardless of who the precise winners are, payments do not change. Hence, it suffices to show that the difference between the total welfare of the bidders in $A$ and that of the bidders in $B$ is bounded above by $k\epsilon$.

Assuming sincere bidding, $S_m \subseteq B$, because demand may fall below $k$ in the $m$th round, so that some bidders in $S_{m-1}$ are allocated. (In the worst case, $S_m = \emptyset$.) Additionally,

• every bidder in $A \setminus S_m$, has value at most $m\epsilon$—otherwise, they would be in $S_m$; and

\(^9\)N.B. The definition of an English auction used in Homework 4 differs from the definition used here. Here bidders can not re-express interest in the good after expressing disinterest; there, they could. This design is more restrictive, and DSIC; the other was less restrictive, and EPIC, but not DSIC.

\(^10\) breaking ties arbitrarily
• every bidder in $A \setminus S_m$ also has value at least $(m - 1)\epsilon$, because bidding is sincere, the auction terminates at round $m$, and the bidders in $A$ are among those with the highest $k$ values.

Therefore, the maximal difference between the total welfare of the bidders in $A$ and that of the bidders in $B$ is:

$$\sum_{i \in A \setminus B} v_i \leq \sum_{i \in A \setminus S_m} v_i \leq (|A| - |S_m|) (m\epsilon - (m - 1)\epsilon) = (|A| - |S_m|) \epsilon \leq k\epsilon.$$

\[\square\]

References

