In the next few labs, you will be competing in a few different prediction markets, culminating in a lab where you will code up bots to compete in a faster paced, digital prediction market. Please try to solve the assigned problems—analytically and/or with Monte Carlo simulations—before coming to that week’s lab.

1 Prediction Markets

A prediction market is a market in which securities are traded based on the predicted outcomes of future events. Prediction markets can be based on the future outcomes of sporting events, elections, or even the weather. They are interesting because they effectively aggregate the beliefs of all participants, so that the market price conveys the collective belief/probability of an outcome.

2 Market with One Decoy

In this first market, we will flip two coins: $C$, the true coin; and $D$, the decoy coin. Both are fair coins. We will then independently and uniformly at random choose one of the two coin flips to reveal to each student in lab.

You, the students, will then begin trading contracts that are worth $100 if the true coin is heads, and $0, if it is tails. Once trading subsides, we will reveal the value of $C$ and $D$, and settle all contracts.

3 Questions

For all questions, assume that you are told “heads”.

1. What is the probability that $C$ is heads?

2. Let’s say you find out that another person was also told “heads”. What is the probability that $C$ is heads?

3. Let’s say you find out that one other person was told “tails”. What is the probability that $C$ is heads?

4. Let’s say you find out that 100000 other people were told “tails”. What is the probability that $C$ is heads?
5. Let $f_H$ be the fraction of other people that are told heads. What are the two most likely values for $f_H$, keeping in mind that you were told heads.

6. How might you infer what other people were told from how they behave in the market?