Lab 9: Combinatorial Auctions, Part 2

Auction Rules

Just like last week, you will again be building agents that bid in combinatorial auctions in today’s labs. We will be using two auction formats this week, both of which are ascending. The first one is very close to the SMRA auction we studied last week, with two main differences: the activity rule and the means of determining the final outcome. The second one allocates entirely differently, limiting exposure. Both games use the same activity rule, the so-called revealed preference rule, described below, and the same means of determining the final outcome.

SMRA*

\[
\text{do } \{
\text{Get bids from each bidder, one per good}
\text{Only keep bids on goods that exceed the good’s current price}
\text{Only keep bids that satisfy the revealed preference rule}
\text{If the set of valid bids is non-empty } \{
\text{For each good } \{
\text{Tentatively allocate to a highest bidder}
\text{Tentatively set the payment to the highest bid}
\}\text{ Notify each bidder of their own allocation, and all prices}
\}\text{ until the set of valid bids is empty}
\text{The final allocation and final payments are set to be the outcome that maximized revenue, maximizing over all rounds.}
\]

This auction differs from the SMRA auction we studied last week in two ways. We describe the activity rule below, but for now, let us briefly comment on the means of determining the final outcome. It suffices to say that the bids in the final round may not be the ones that yield the highest revenue (or welfare), because some bidders may drop out if what they thought was their preferred bundle ultimately becomes too expensive. In such cases, it is useful to give the auctioneer the freedom to revert back to an earlier allocation and payments.

Combinatorial Clock-like Auction (CCA-like)

\[
\text{do } \{
\text{Get demand sets from all the bidders. That is, each bidder should report its favorite bundle at current prices, assuming prices are as given, if the bidder is currently winning the good, or the current price plus } e, \text{ if not.}
\]
Only keep bids that satisfy the revealed preference rule
If the set of valid bids is non-empty {
    Run winner determination to find a revenue-maximizing allocation
    Increment prices on all goods that changed hands
    Notify each bidder of their own allocation, and all prices
} until the set of valid bids is empty
The final allocation and final payments are set to be the outcome that maximized revenue, maximizing over all rounds.

This auction design is based on Porter et al.’s combinatorial clock auction.\textsuperscript{1} The final allocation and payment rule is inspired by Ausubel et al.’s clock-proxy auction.

Activity Rule

The revealed preference rule is defined as follows: If a bidder switches its preferred demand set from bundle $S$ to bundle $T$, then it must have been that since the time when $S$ was preferred, the price of $T$ increased by less than the price of $S$.

Note that this rule is applicable in SMRA, by simply mapping an agent’s bids, each of which names a good and a price, to a preferred bundle and a corresponding price. This bundle contains all the goods on which an agent bid, with a price equal to the sum of the goods’ prices.

Here is a formal statement of the revealed preference rule: Let $s < t$ be two rounds in the clock phase. Let $p^s$ and $p^t$ be vectors in $\mathbb{R}^m$ describing good prices at rounds $s$ and $t$, respectively. Let $x^s$ and $x^t$ be vectors in $\{0,1\}^m$ describing a bidder’s demands at rounds $s$ and $t$, respectively. If a bidder is bidding sincerely, then at round $s$, $x^s$ is the set of goods that is utility maximizing:

$$v(x^t) - p^t \cdot x^t \geq v(x^s) - p^s \cdot x^s$$

Similarly, at round $t$, $x^t$ is the set of goods that is utility maximizing:

$$v(x^t) - p^t \cdot x^t \geq v(x^s) - p^s \cdot x^s$$

Combining these inequalities yields the revealed preference rule:

$$(p^t - p^s) \cdot (x^t - x^s) \leq 0$$

\textsuperscript{1} Many countries have used CCAs to sell radio spectrum licenses, including Australia, Austria, Canada, Denmark, Ireland, the Netherlands, Slovenia, Slovakia, Switzerland, and the United Kingdom.
This rule is intended to encourage sincere bidding. For example, it encourages bidders to not take a break from bidding part way through the auction, because if they do (i.e., if they report the empty set as their favorite bundle), and then if prices go up on some goods, then they can no longer bid on bundles with those goods, because the prices of the (nonexistent) goods in the empty set could not possibly have increased by more than the prices of the actual goods whose prices increased. In particular, this rule discourages bidders from sitting out the very first round.

However, the revealed preference rule does not ensure that bidders bid sincerely, especially in SMRA*. With item rather than bundle prices, and combinatorial valuations, bidders still must strategize about how to bid. For example, valuations and prices could be such that

\[ v(S) < p(S) < p(T) < v(T), \]

where \( S \subseteq T \). In this case, winning only \( S \), but not losing bundle \( T \setminus S \) would be disastrous.

**Implementation**

In this lab, you will be writing a bidding agent that extends Lab8Agent (again). Refer to [Lab 8](#) for details about the API.

**Parameters**

Here are the relevant parameter settings for today’s games:

- Your initial valuations will consist of some number of atomic bids drawn from a normal distribution with mean 50 and standard deviation 15.
- You will have 5 seconds to submit your bids (i.e, each round of each of the ascending auctions will last 2 seconds).
- The bid increment (\( \epsilon \)) is set to 20.