Normal-Gamma

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$p(x|\mu,\sigma^2) \Rightarrow$ use precision $\frac{1}{\sigma^2} \Rightarrow p(x|\mu,\tau)$

$$p(x|\mu,\tau) = \sqrt{\frac{\tau}{2\pi}}\, e^{-\frac{\tau}{2}(x-\mu)^2}$$

Likelihood ~~given~~ of Data $x_1 \ldots x_n$

$$p(D|\mu,\tau) = \prod_{i=1}^{n} \sqrt{\frac{\tau}{2\pi}}\, e^{-\frac{\tau}{2}(x_i-\mu)^2}$$

~~we're~~ conjugate Prior

$$p(\tau|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau} \Leftarrow \text{Gamma}(\alpha,\beta)$$

posterior $p(\tau|D,\alpha,\beta,\mu) \propto p(D|\mu,\tau)\,p(\tau|\alpha,\beta)$

we're using conjugacy, so multiplicative constants can be dropped

$$p(\tau|D,\alpha,\beta,\mu) \propto \tau^{n/2} e^{-\frac{\tau}{2}\sum_i (x_i-\mu)^2}\, \tau^{\alpha-1} e^{-\beta\tau}$$

$$= \tau^{\alpha+n/2-1} e^{-\tau(\beta+\frac{1}{2}\sum_i (x_i-\mu)^2)}$$

we know this form & $\therefore$ the normalization

$$= \text{Gamma}\left(\alpha+n/2,\ \beta+\frac{\sum_i (x_i-\mu)^2}{2}\right)$$

always goes up!

goes up by $\sum$ 2nd dist to mean/2 (maybe not so much)