ENGN 2520 / CSCI 1950-F Homework 6
Due Tuesday April 16 by 4pm

Students may discuss and work on homework problems in groups. However, each student must write down their solutions independently.

All of the work submitted should be your own. NO COPYING from ANY source is allowed.

Each student should write on the problem set the set of people with whom they collaborated.

Problem 1

(a) Let H denote the set of axis-parallel rectangles in the plane. That is, each $h \in H$ is defined by a range of x values and a range of y values. Each rectangle defines a binary classifier that assigns label $+1$ to points inside the rectangle and label -1 to points outside the rectangle. Show that the VC dimension of H is 4.

(b) Now suppose the input space consists of points in $\mathbb{R}^n$ and H is the set of axis-parallel hyper-rectangles. What is the VC dimension of H? justify your answer.

(c) Suppose we want to learn an axis-parallel hyper-rectangle concept in $\mathbb{R}^{10}$. We will do this by finding a hypothesis that is consistent with m training examples. Suppose we want a hypothesis that has probability of error at most 0.1 with probability at least 0.99. How big should we make m?

Problem 2

Let X be an input space and H be a finite set of functions from X to $\{-1, +1\}$. Show that the VC dimension of H is at most $\log_2 |H|$.

Problem 3

(a) Let A and B be two sets of binary classifiers over the same input space. Show that if $A \subseteq B$ then the VC dimension of A is bounded by the VC dimension of B.

(b) Let X and Y be two input spaces, ϕ be a map from X to Y and H be a set of binary classifiers over Y. For each $h \in H$ we can define a classifier h' over X by first mapping points in X to points in Y, and then classifying the resulting points using h. That is

$$h'(x) = h(\phi(x))$$

Let H' be set of all such classifiers. What can you say about the relationship between the VC dimension of H and H'? Justify your answer.

(c) Let Q be the set of classifiers over $\mathbb{R}^n$ defined by thresholding a polynomial of degree at most 2 of their inputs. Show that there is a positive integer m and a map $\phi : \mathbb{R}^n \to \mathbb{R}^m$ such that for every $q \in Q$ there is a linear threshold function h over $\mathbb{R}^m$ such that $q(x) = h(\phi(x))$.

(d) Use the results of parts (a), (b) and (c) together with what we know about linear threshold functions to bound the VC dimension of Q.