**Problem 1.** Show that if a sequence $X = x_0, \ldots, x_{w-1}$ is $k$-smooth, then the result of passing $X$ through a balancing network is $k$-smooth.
Problem 2. Let $\mathcal{B}$ be a width-$w$ balancing network of depth $d$ in a quiescent state $s$. Let $n = 2^d$. Prove that if $n$ tokens enter the network on the same wire, pass through the network, and exit, then $\mathcal{B}$ will have the same state after the tokens exit as it did before they entered.
**Problem 3.** Consider the following code for an in-place merge-sort:

```java
void mergeSort(int[] A, int lo, int hi)
{
    if (hi > lo)
    {
        int mid = (hi - lo)/2;
        executor.submit(new mergeSort(A, lo, mid));
        executor.submit(new mergeSort(A, mid+1, hi));
        awaitTermination();
        merge(A, lo, mid, hi);
    }
}
```

Assuming that the merge method has no internal parallelism, give the work, span, and parallelism of this algorithm. Give your answers both as recurrences and as $\Theta(f(n))$, for some function $f$.
Problem 4. Professor Jones takes some measurements of his (deterministic) multithreaded program, which is scheduled using a greedy scheduler, and finds that $T_4 = 80$ seconds and $T_{64} = 10$ seconds. What is the fastest that the professor’s computation could possibly run on 10 processors? Use the following inequalities and the bounds implied by them to derive your answer. Note that $P$ is the number of processors and $T_p$ is the time taken with $P$ processors.

\begin{align*}
    T_P &\geq \frac{T_1}{P} \quad (0.1) \\
    T_P &\geq T_\infty \quad (0.2) \\
    T_P &\leq \frac{(T_1 - T_\infty)}{P} + T_\infty \quad (0.3)
\end{align*}

(The last inequality holds on a greedy scheduler.)
Problem 5. (Bitonic Counter Programming Assignment: WIP)