

Homework 3

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Problem 1. Consider a system with an object x and n threads. Determine if each of these properties are equivalent to saying x is deadlock-free, starvation-free, obstruction-free¹, lock-free, wait-free, or none of these. Briefly justify your answers.

1. For every infinite history H of x , an infinite number of method calls are completed.
2. For every finite history H of x , there is an infinite history $H' = H \cdot G$.
3. For every infinite history H of x , every thread takes an infinite number of steps.
4. For every infinite history H of x , every thread that takes an infinite number of steps in H completes an infinite number of method calls.
5. For every finite history H of x , there are n infinite histories $H'_i = H \cdot G_i$ where only thread i takes steps in G_i , where it completes an infinite number of method calls.
6. For every finite history H of x , there is an infinite history $H' = H \cdot G$ where every thread completes an infinite number of method calls in G .

Problem 2. This problem examines a stack implementation (Figure 1) whose `push()` method does not have a single fixed linearization point in the code.

The stack stores its items in an `items` array, which for simplicity we will assume is unbounded. The `tail` field is an `AtomicInteger`, initially zero. The `push()` method reserves a slot by incrementing `tail`, and then stores the item at that location. Note that these two steps are not atomic: there is an interval after `tail` has been incremented but before the item has been stored in the array.

The `pop()` method reads the value of `tail` and then traverses the array in descending order from the `tail` to slot zero. For each slot, it swaps `null` with the current contents, returning the first non-`null` item it finds. If all slots are `null`, the method returns `null`, indicating an empty stack.

- Give an example execution showing that the linearization point for `push()` cannot occur at Line 11. Hint: give an execution where two `push()` calls are not linearized in the order they execute Line 11.
- Give another example execution showing that the linearization point for `push()` cannot occur at Line 12.
- Since these are the only two memory accesses in `push()`, we must conclude that `push()` has no single fixed linearization point. Does this mean `push()` is not linearizable?

Problem 3. For each of the histories shown below (figures 2 and 3), are they sequentially consistent? Linearizable? Justify your answer.

¹An implementation is *obstruction-free* when any thread will complete a method call if it is given some finite number of steps in isolation. Note that a step is one of many (possibly infinite) instructions within a method call.

```

1 public class AMGStack<T> {
2     AtomicReferenceArray<T> items;
3     AtomicInteger tail ;
4     static final int CAPACITY = 1024;
5
6     public AMGStack() {
7         items = new AtomicReferenceArray<T>(CAPACITY);
8         tail = new AtomicInteger(0);
9     }
10    public void push(T x) {
11        int i = tail.getAndIncrement();
12        items.set(i,x);
13    }
14    public T pop() {
15        int range = tail.get();
16        for (int i = range - 1; i > -1; i--) {
17            T value = items.getAndSet(i, null);
18            if (value != null) {
19                return value;
20            }
21        }
22        // Return Empty.
23        return null;
24    }
25 }

```

Figure 1: Afek/Morrison/Gafni stack.

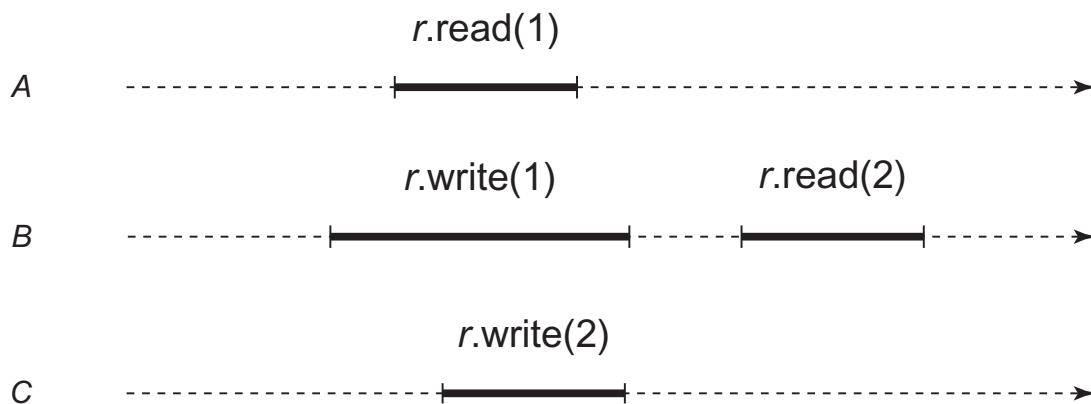


Figure 2: First history for Exercise 3.

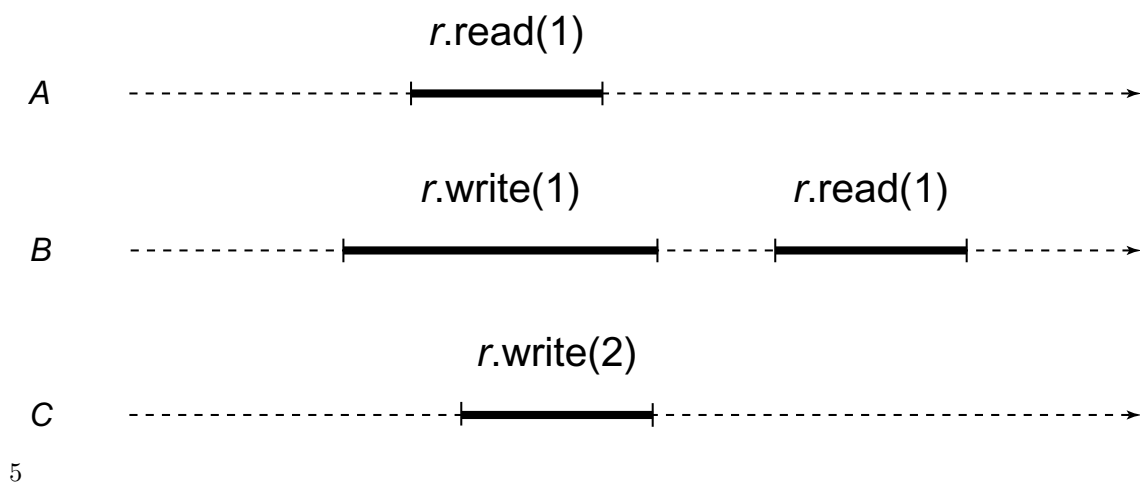


Figure 3: Second history for Exercise 3.