This is a partner homework as usual: work on all problems together; you are responsible for everything you and your partner submit and it is an academic code violation to submit something that is not yours. Check for your partner in the Google doc that was sent out. If you don’t have a partner, please let us know.

Each of the 5 problems in this assignment may be turned in separately, for a separate deadline. Run cs157_handin hw7-p3 to copy everything from your current directory to our grading system as a submission for problem 3. (Change “3” to a number 1 to 5 for the other problems!)

Throughout this assignment you will be finding inputs to optimize (minimize) the output of certain functions. Each function can be found in /course/cs157/pub/stencils/hw7, along with stencil code and helper code. Each of these functions, in addition to outputting a number, will also graphically display its result (at most a few times per second, to give you some feedback without spending too much time rendering graphics). Pay attention to the graphics! This is the only way to get intuition for what is going on in this assignment. One good insight can save you hours of execution time! All the code for this assignment is at your fingertips, so also consider playing around with the starter code to get it to report different things, etc.

See the section on Matlab at the end of this document for reminders about helpful Matlab features. Matlab’s graphics will occasionally crash; if this happens, press Ctrl-C to stop execution, and type close all to reset the graphics.

Problem 1

Local search:

This problem will get you started with some code that you can use for the next three problems.

In this problem you will be writing a local search optimization routine, along with a few different routines for generating “local proposals”. Local search, when given a function $f$ to minimize, and a current input $x$, repeatedly tries to modify $x$ so as to decrease $f(x)$. Each of these modifications is generated by a local proposal function, that, when given $x$, proposes a nearby $x'$. The simplest kind of local search accepts proposals that decrease $f$, and rejects proposals otherwise. As it turns out, it is often reasonable to allow “neutral” moves, that is, moves that do not affect the function value; further, it is even useful to allow slightly harmful moves, for example, for a parameter $\epsilon$, accepting proposals that increase $f$ by at most $\epsilon$.

For this part, turn in code as described below. Document anything that needs clarification, though this part will not need much.

1. Fill in the code localSearch. In particular, we suggest you make use of the Matlab now function which returns the current date and time as a real number, in units of days (namely, comparing now*60*60*24 at two different moments will will tell you how many seconds have
passed in between; this will help you write a stopping criterion. This function will not be runnable until you write one of the proposal functions below.

You will run this code with \textit{function handles} as inputs, namely the symbol “@” before a function name. Many optimization problems (including problem 3 below) come with bounds on their inputs, and it is useful to make sure the proposals obey these bounds; thus your code will have parameters \texttt{lowerBoundOnX} and \texttt{upperBoundOnX} that will help ensure that proposals are valid.

\textbf{(Hint:} You will find it very useful, later, for your \texttt{localSearch} function to output information about its partial progress. In particular, perhaps every time a proposal is accepted, report the value $\texttt{func(x)}$ to the command window—by just putting this statement in the code, without a semicolon at the end.

2. Fill in the code \texttt{wideScaleRandomNoiseProposal}, which chooses a radius from a wide range, at least $[0.0001, 100]$, and modifies each coordinate of the input by the radius times a randomly chosen positive or negative number (use \texttt{randn} instead of \texttt{rand} to get negative—actually, normally distributed—numbers). Make sure the radius is chosen so that its logarithm is uniformly distributed (that is, choose a random number uniformly, using \texttt{rand}, and set radius to be 10 to the power of this random number). This kind of function will efficiently propose both small and large changes. (Can you intuitively see why?)

Note: the Matlab command \texttt{size} returns the size of each dimension of a Matlab variable, which may be useful for this code; alternatively, \texttt{numel} returns the number of elements in the variable, which is equivalent to the product of the dimensions.

3. Test your code from above. \textbf{Note:} Only part \texttt{c} needs to be turned in.

(a) Easy case: Try the 2-dimensional “horseshoe” function from class, optimized for 3 seconds (the parameter \texttt{inf} at the end means infinity, and should make your \texttt{localSearch} function cut off after exactly 3 seconds, according to the second to last parameter):
\begin{verbatim}
localSearch(@g5,@wideScaleRandomNoiseProposal,[0 0],0,-10,10,3,inf);
\end{verbatim}

(b) Medium case: Try to optimize the (square of the) distance from the origin in 100-dimensional space, starting far from the origin. (Yes, we all know the optimum is at the origin, but does your proposal function let your local search routine find the origin efficiently?) Your routine should fairly quickly converge to value of around $1e-5$ (corresponding to distance 0.003 from the origin); if not, try to see why, intuitively, your local search routine is not making and accepting good proposals, and then go back to previous parts and make sure you are implementing the right thing.
\begin{verbatim}
x0=1000*randn(1,100);
localSearch(@(x)sum(x.^2),@wideScaleRandomNoiseProposal,x0,0,-inf,inf,10,inf);
\end{verbatim}

(c) Hard case: Fill in the stencil \texttt{absMinimize} to minimize the sum of the absolute values function $f(x)=\text{sum(abs(x))}$, in 100 dimensions. While this is a convex function, and seems like it should be well-behaved, high dimensional geometry plays tricks on us. If, as above, you repeatedly search for local proposals that will improve the function value, you will very quickly find yourself in a deep narrow valley, where a small fraction of the 100 coordinates are large, the rest are near zero, and further progress downhill becomes vanishingly improbable. (To see this, try setting \texttt{epsilon} to a vector with several zeros, as in \texttt{epsilon=[0 0 0 0]} in the stencil, and run it.) To fix this, we need to try larger values of epsilon, which is counterintuitive: $\epsilon > 0$ means that local search will
accept proposals that are worse than the current situation. Despite how strange this sounds, having larger values of epsilon emphasizes exploration instead of exploitation—when epsilon was 0, the search was getting stuck, so we need to make epsilon larger, to let local search get unstuck and explore more effectively. Your task here is to design a “schedule” for the epsilons, that starts with something high, and gradually decreases epsilon so that the function optimizes effectively. You know that epsilon is too small if the search freezes into a “spiky” configuration like you saw for $\epsilon = 0$; however, by the end of the schedule of epsilons, the last epsilon should be small enough for the function to optimize to something less than 1. See what intuition you can build about how the shape of this “cooling schedule” affects the optimization. In the problems below, see if clever choices of epsilon can help get you around obstacles.

4. In some cases modifying all the coordinates at once may be too drastic. Fill in the code `wideScaleRandomNoiseOneCoordinateProposal`, which functions exactly as above, except it only modifies a single entry of the input, chosen at random.

5. In problem 2 below, the proposal of the previous part will have the effect of modifying either the $x$ or $y$ coordinate of a random disk, though what you really want to do is modify both coordinates of a random disk. In this part, fill in the code `wideScaleRandomNoisePairProposal`, which chooses a random consecutive pair of coordinates, $(1, 2)$ or $(3, 4)$ or $(5, 6)$ or ... and modifies these entries randomly as above.

6. Sometimes many different types of proposals may all help. In this problem fill in the code `wideScaleRandomNoiseMix3Proposal` which picks a random one of the previous three functions and calls it to generate a proposal.

You can test your code for this section before moving onto the next section with the function $g_5$ which is one of the functions we optimized in class. For example, run

```localSearch(@g5,@wideScaleRandomNoiseProposal,[0 0],0,-10,10,10,0.01);```

Note that because $g_5$ is a two-dimensional function, `wideScaleRandomNoiseProposal` and `wideScaleRandomNoisePairProposal` should have identical effect.

**Problem 2**

**Circles:**

In this problem you will be minimizing the output of the function `arrangeCircles`, which attempts to arrange nonoverlapping disks of radii 1, 2, 3, ..., 10 so that they fit into the smallest square. Read the description written in the comments at the top of the file to figure out what the function does, and try running it with example inputs, such as `arrangeCircles(1:20)`. Your job is to minimize the output of this function.

You cannot expect to find the exact optimum of this function, but the methods of the previous part will get you quite far. Potential things to try include: play with the value of epsilon in the `localSearch` function; try restarting the search if it does not make progress (and perhaps modify `localSearch` to output debugging/partial progress information); try creating new proposal functions; if you have intuitions about which direction to modify an arrangement, you can try encoding your intuitions as hints to the local search routine via a modification of the `arrangeCircles`
function—though keep in mind that your results will of course be evaluated on an unmodified arrangeCircles function.

Turn in:

1. The best arrangement of circles you have found, stored as a variable x in a file bestcircles.mat, via the Matlab command

   save bestcircles x

2. Runnable code that you used to generate x above, including a main routine arrangeCirclesRunner that calls (presumably) the localSearch function with a specially chosen set of parameters, along with anything else you found useful. Important: For this and subsequent problems, be sure to turn in LocalSearch along with any proposal functions it needs—even if you already turned them in for Problem 1. The course staff needs to be able to run your submission for this problem in order to grade it.

3. The majority of your grade will come from the explanation found in the comments of arrangeCirclesRunner.

Your grade will be 40% from the quality of the solution x you found (in this problem, 41 is a good square size to aim for); 60% of your grade will come from the writeup, which should demonstrate an understanding of how you achieved the performance you did, explain any unexpected choices you made, and explain any innovations in your code. Unlike in previous assignments where everyone’s code was supposed to be identical, this assignment encourages creativity and exploration; for us to be able to grade it, you need to document your code well. Points will be taken off if we cannot figure out how your code works. Note that because you will be using randomized algorithms on this assignment, running the same code twice will produce different results; nevertheless, you should aim to write code that is good and not just lucky, and your writeup should justify this. In principle you could run your code from now until the assignment is due and report the best result; but in such cases, we will not be able to re-run your code to verify your results, and you must rely on your writeup to convince us of your code’s quality. Of course, code that accomplishes the same task, reliably, in less time is preferred.

Problem 3

“Twop”:

Optimization is amazing. You can write code to solve unusual, unexpected, complicated problems that you would not know how to solve yourself. In this problem you will try your hand at one of the legendary problems of robotics (and animorphs): moving around effectively on two legs. To give you a sense of how hard this problem is, try the Flash game at [http://www.foddy.net/Athletics.html](http://www.foddy.net/Athletics.html) known as “QWOP”.

In this problem, you will be using optimization code to help Tobias, the quiet loner of the Animorphs who was trapped as a hawk, learn how to move again as a human. You have a simulation of QWOP written in Matlab, called twop, that takes as input a sequence of 20 pairs of commands to Tobias’ legs, and you will need to write code to figure out how to get Tobias as far forward as possible before hitting the ground or reaching the end of the 20 commands.
The input to \texttt{twop} is a sequence of 40 numbers, interpreted as 20 pairs of numbers, each between \(-1\) and \(1\). The first number in each pair represents the force on Tobias’ thighs: positive brings one leg forward, negative brings the other leg forward. The second number in each pair represents the force on Tobias’ calves (the lower part of his legs): positive brings one foot forward, negative brings the other foot forward. Try a few example inputs:

\texttt{twop(rand(2,20)*2-1)}

Note that if \texttt{twop} is called from the command line, it will display an animation, but if it is called from within a function, it will display a “time–lapse” display of his trajectory. (Feel free to edit \texttt{twop} if you want to tweak this behavior.)

The return value of \texttt{twop} is negative the \(x\)-coordinate of Tobias’ hip when all 20 movements have been processed, or when his hip or head hits the ground, whichever happens first. (As usual, we are minimizing the result of this function, hence the negative sign; Tobias still wants to get as far to the right as possible.)

This problem is rather open-ended, and there are many things you could try to improve Tobias’ performance. Intuitively, there is something sequential about this optimization problem: unless Tobias starts out well, he will not get far. Bad starts for Tobias include hitting his head early. Good starts might include flying upright at high speed to the right.

As explained in more detail for problem 2 above, turn in:

1. (40\% of the points) The best parameters you have found, getting Tobias as far to the right as possible, stored as a variable \(x\) in a file \texttt{besttwop.mat}, via the Matlab command

   \texttt{save besttwop x}

   Good performance is reaching location 8; getting past 9 is impressive.

2. Runnable code that you used to generate \(x\) above, including a main routine \texttt{twopRunner} that calls other optimization routines you wrote.

3. The majority of your grade will come from the explanation found in the comments of \texttt{twopRunner}.

Problem 4

“Traveling Salesman”:

In this problem you will write optimization code for the traveling salesman problem (NP-hard!). Locations of 100 American cities (latitude and longitude) are stored in the Matlab data file “cities.mat”. (You can load this by typing \texttt{load cities}; you could play around with the data, or possibly even use it directly in your optimization code, though this is not necessary.)

The input to the function \texttt{travelingSalesman} is an order in which to visit the 100 cities, namely a permutation of the numbers 1 to 100. Try running this with the trivial ordering: \texttt{travelingSalesman(1:100)}.

The return value of this function is the total distance traveled: if your permutation starts 57, 12, 34, ..., then the total distance is the distance between cities 57 and 12, plus the distance between cities 12 and 34, etc. Your goal is to minimize this function.
Note that “wide scale random noise” is not an appropriate way to locally search the input space, since inputs must have a very particular form. Inputs must consist of exactly one copy of each of the numbers (cities) 1 through 100, in some order. Thus your challenge is to think of reasonable local proposals that transform valid permutations into other valid permutations. One example that was suggested in class (try this!) is to pick two random cities and swap them in the list. See how this works; see if it seems to be missing “obvious” things to try; come up with new proposals on your own; once you have some ideas for proposal functions, try a random mix of your best ideas!

As explained in more detail for problem 2 above, turn in:

1. (40% of the points) The best parameters you have found, describing the shortest tour of the 100 cities (expressed as a permutation of the numbers 1 through 100), stored as a variable \( x \) in a file `bestsalesman.mat`, via the Matlab command

   ```matlab
   save bestsalesman x
   ```

   A good value to aim for is 350. The best we have seen is 327.

2. Runnable code that you used to generate \( x \) above, including a main routine `salesmanRunner` that calls other optimization routines you wrote.

3. The majority of your grade will come from the explanation found in the comments of `salesmanRunner`.

**Problem 5**

**Image Denoising:**

This problem is very different from the others on this assignment. While you will be writing code that optimizes a function, here you must not use local search.

Since cheap digital cameras are everywhere, an important problem that has many applications and generalizations is “image denoising”. Namely, given an image where the value at each pixel has been corrupted by the addition of a random number, is there any way to recover the original image? (This corruption occurs at the hardware level of cameras, as cheap electronics cannot accurately read out the intensity of light at a given place on the camera sensor; the “original image” means the intensity of light at each gridpoint on the sensor, and it only ever reaches your computer in corrupted form.)

The language of optimization provides a very powerful framework to express different approaches to denoising. The general intuition behind denoising is that that, given an input image \( \text{inp} \), we want to output an image \( x \) that balances two things: 1) \( x \) is “less noisy” than \( \text{inp} \), and 2) \( x \) is “close to” \( \text{inp} \). The reason for the second component is that otherwise optimization would return an all-gray image, which is certainly less noisy, but has nothing to do with the original image. Any way we choose to define the “noisiness” of \( x \), and the “closeness” of two images \( x \) and \( \text{inp} \) will give us a different notion of denoising!

In this problem we will consider quadratic (L2) denoising. In lecture we will see L1 denoising, which is often much better, but much harder to implement. The function you will optimize is defined as the sum of the following two terms:
1. The sum over each pair of horizontally or vertically adjacent pixels of the square of the difference between their intensities in the image $x$, multiplied by a parameter $k$.

For reasons which will become clear when you solve this problem, the notion of adjacent pixels here should also include: each pixel in the top row is adjacent to the corresponding pixel in the bottom row, and each pixel in the leftmost column is adjacent to the corresponding pixel in the rightmost column.

2. The sum over each pixel of the square of the difference between its intensity in $x$ and its intensity in the input image $\text{inp}$.

That is, you should write a function $\text{denoiseQuadratic}$ that takes in an input image $\text{inp}$ (a 2-dimensional array; no color information), and a positive parameter $k$, and outputs an image $x$ that minimizes the above function as computed on $x$, $\text{inp}$, and $k$.

However: You should not write optimization code. This problem is solvable with the magic of Fourier transforms in time $O(N \log N)$ for an image with $N$ pixels. For other notions of denoising, this performance is impossible, but for quadratic denoising, this happens to work out, and your challenge is to figure out how.

One crucial fact about Fourier transforms: the sum of the squares of the magnitudes of $x$ equals the sum of the squares of the magnitudes of its (1 or 2-dimensional) Fourier transform (possibly times a scaling factor, dependent on how the Fourier transform is defined, which does not matter).

This is known as Parseval’s theorem (in particular, for discrete Fourier transforms, see the last equation on the Wikipedia page.) We emphasize “magnitude” here because the entries of a Fourier transform may be complex numbers, and taking their magnitude gives us nice real numbers.

Warning: be very careful about what operations are done pixel-by-pixel on the image, vs. across different pixels.

The final result of your code will look like a blurred version of the original image, though the question is which blurring. Non-quadratic notions of denoising can yield much more subtle results than just blurring, but they take a lot more work to compute.

(Hints: This problem makes crucial use of convolution. The best way to do this problem is to work out as much of the math as you can with pencil and paper before asking the computer to do anything. The last step, conceptually, is to minimize a quadratic function of one variable. Bear in mind that, due to rounding errors, some of the results may have unexpected imaginary components, so you might need to use the Matlab function $\text{real}$ on your answers so that plotting routines do not crash.)

You can load a test image with the commands $\text{inp=mean(imread('theunknown.jpg'),3)}$;

To plot an image $x$, type $\text{imagesc(x)}$ (do not use the similar $\text{image}$ function; $\text{imagesc}$ scales the colors sensibly first, which is useful). To make the colors grayscale, type $\text{colormap gray}$. Matlab will complain if there are imaginary numbers in your input. You can plot the real part of $x$ with $\text{imagesc(real(x))}$.

** If you load images another way, make sure your input is double precision, and not an integer data type, which will lead to numerical issues. Also, make sure the input to your function is a matrix (2-dimensional), not 3-dimensional. By default Matlab loads red, green, and blue color components along 3 slices in the 3rd dimension; the command above takes the mean along the third dimension to recover the average, grayscale value of the image.
** Make sure you are using the two dimensional versions of fft and ifft, which are fft2 and ifft2!

** You do not need to turn in any explanation or writeup for this problem. However, if your code has bugs or is wrong, then we need an explanation or writeup so that we can give you partial credit. You can add comments to your code, scan or take a picture of your pencil+paper work and submit it with your code, etc.

Test case: If you load the test image with the above `inp=mean(imread('theunknown.jpg'),3)` command, then run your code with $k = 1$ as $x=\text{denoiseQuadratic}(\text{inp},1)$, then you can check the objective value of the function you are trying to minimize by running the code `\text{denoiseQuadraticTest}(\text{inp},1,x)` that we are providing for you. If you have optimized $x$ correctly, the value for the test image should be $1.1545e+008$.

Matlab hints:

The Matlab command `dbstop if error` sets Matlab to go into debug mode any time there is an error, or you interrupt execution with `Ctrl-C`. The command prompt will change to “K>>”, to indicate that Matlab is in the middle of code execution. You can also reach debug mode by putting breakpoints in your code. You can see the call stack by typing `dbstack`, and can move up and down the call stack by typing `dbup`, or `dbdown`. The easiest way to transfer variables to the base workspace is with the Matlab `save` and `load` commands, which saves variables to a specified `.mat` file on disk, and lets you load them later. To quit debugging mode, type `dbquit`; otherwise, you might get two or three levels deep in debugging mode, which can confuse you and Matlab.

To profile your code for speed, type `profview`, enter code to run in the code box, and click around.

As mentioned at the start, Matlab’s graphics may occasionally freeze, in which case you should stop execution with `Ctrl-C`, reset the windows with `close all`, and, if the command prompt is `K>>`, type `dbquit`. Note that it is advisable to write your optimization code to output enough information about partial progress so that you can tell whether or not it is stuck.